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'STATISTICAL CHARACTERIZATION OF ALTITUDE MATRICES BY COMPUTER'

REPORT 4

Frequency distributions of gradients

THE FOURTH PROGRESS REPORT ON GRANT DA-ERO-591-73-G0040

By IAN S. EVANS, M.A., M.S., PH.D., (Principal Investigator)
Department of Geography, University of Durham, England

To DR. H. LEMONS
Chief Scientist, European Research Office, U.S. Army

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Statistical characterization of altitude matrices by computer.

REPORT 4.

FREQUENCY DISTRIBUTIONS OF GRADIENT.

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CONTENTS

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(12) 56.

FREQUENCY DISTRIBUTIONS OF GRADIENT

ABSTRACT

Gradient is the most important attribute of surface geometry and its frequency distribution is considered here in detail to assess how it may be summarised, for example by fitting various models. Plots on probability paper are made of gradients (i) from altitude matrices of 25 to 100m mesh, for five square areas and for two drainage basins, (ii) from meshes of variable triangles averaging 33 to 244m in linear dimension, for five drainage basins, (iii) from relief per 1 x 1 km square for large morphological regions, and (iv) from field measurements over distances of 1.5 to 10m along profiles, the location of which was subjective. Although some support is provided for Speight's (1971) suggestion that taking the logarithm of tangent normalises frequency distributions, in some cases better results are obtained from the square root of sine, or even from no transformation of slope angle in degrees. The main transformations have similar effects over a broad range of gradients, and most existing data sets are insensitive to the difference between them. But the differences which are found here are probably due to differences in terrain, more than the use of different measuring techniques or differently-defined study areas. Skewness, for example, does not vary drastically with grid mesh. Hence the tentative conclusion is that even if study areas are comparably defined, and identical techniques are used, there is no single universally applicable transformation which normalises gradients. Summarisation of gradients over an area for the purpose of comparison with other areas therefore requires skewness and kurtosis as well as mean and standard deviation. The simplest approach is to calculate these four moment-based statistics for gradient expressed in degrees, but it may be useful to go on to further calculations on whatever transformed scale is found appropriate.

FREQUENCY DISTRIBUTIONS OF GRADIENT*

PREVIOUS WORK If an area is to be described by summary statistics of point values such as gradient, it is important to establish the shape of the statistical frequency distribution of such values. Evans (1972), in proposing the use of moment-based summary statistics, assumed that values for gradient would not in general follow the normal frequency distribution model. He proposed, therefore, that skewness and kurtosis were required to supplement mean and standard deviation as descriptors of the gradient frequency distribution. For slope profile data measured in the field, however, Pitty (1970) considered the problem of outliers sufficiently disturbing that equivalent percentile-based measures should be used instead. Such additional statistics would not be required if frequency distributions followed some single model, not necessarily the normal model. In the case of gradient values several such models have been proposed. Strahler (1950) stated that:

> "Within an area of essentially uniform lithology, soils, vegetation, climate and stage of development, maximum slope angles tend to be normally distributed with low dispersion about a mean value determined by the combined factors of drainage density, relief and slope-profile curvature."

This proposal, based on observations in the steep-sided valleys of the Verdugo and San Raphael Hills, southern California, is stated sufficiently precisely that the limitations to its application are clear. It applies only to the maximum angle in each slope profile, and only to areas of rare homogeneity. Later, Strahler (1956) used the sine of slope angle; this was endorsed by Tricart (1965, p.166), but Miller and Summerson (1960) and Mayr (1973) preferred the square root of the sine. Thomas and Tuttle (1967) used a logarithmic transformation of the tangent of gradient, before applying significance tests. Blong (1975) chose the tangent of gradient, but did not

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*GRADIENT is defined here as the maximum rate of change of altitude at or around a point on the land surface. Unless otherwise stated it is expressed in degrees, rather than as a tangent. Gradient is only one component of slope, which also includes aspect, the direction of maximum rate of change of altitude.

demonstrate what improvement was achieved thereby.

The first attempt to compare different transformations of slope frequency distributions was by Speight (1971). He compared logarithm, square root and no transformations, of tangent, angle and sine for gradient data collected in different ways by Seret (1963), Young (1961), de Béthune and Mammerickx (1960), Strahler (1956), Gregory and Brown (1966) and himself. The clearest conclusion was that, except for some of Strahler's data, transformation was required to reduce the general positive skew. The difference between logarithmic transforms with slight negative skew (least skewed for log tangent) and square root transforms with slight positive skew (least skewed for root sine) was not marked (e.g. Speight 1971 Fig.1, for Seret's data). Speight decided that log tangents had the advantage, but for some areas it was advisable to fit steep and gentle slopes by different log-tangent normal models, e.g. the Bougainville and Buka Islands and the McArthur R. area. Strahler's data were strongly negatively skewed on the log-tangent scale, and normal curves could be fitted only by ignoring gentler slopes.

The difficulty of discriminating one transformation from another is shown by Fig.1. Whether the logarithm of angle or of tangent is taken, no difference can be established below 20°; the relationship between the two transformations is linear. Only above 50° does the plot curve appreciably, but none of the available data sets has as much as 1% of its gradient in that range. More to the point, Fig.2 relates the two best transformations (of those considered by Speight), the square root of sine and the logarithm of tangent. The relationship is very close to linear between 10° and 50°. Given the rarity of steeper slopes, discrimination between the two transformations can be achieved only in terms of gentle slopes, preferably below 5°. Clearly it would be useful to subdivide the 'below 1°' class.

Speight demonstrated how the planimetry of facets from morphological maps by Seret (1963) and Gregory and Brown (1966) exaggerated minor modes; it is necessary to smooth such data. Speight found little evidence of the polymodality ('characteristic slope angles') which several authors had seen in their data. Clearly such characteristics must be judged on the transformed measurement scale. Nevertheless, Speight's technique of plotting the ratio of observed to expected values is not adopted here, since it exaggerates the importance of small numbers in peripheral classes.

Nieuwenhuis and van den Berg (1971), in a paper notable for its recognition of autocorrelation in slope profile data, applied a square root transformation to tangent data and suggested that this resulted in insignificant deviation from the normal frequency distribution model. Unfortunately, as they admitted on p.167, their slope profiles were subjectively located : hence, despite the careful thinning out to eliminate significant autocorrelation, their application of significance tests permits conclusions only about the particular profiles chosen, and not about the study area. They failed to make the necessary qualifications to their conclusions, e.g. on p.172 and in the abstract. They demonstrated on p.170 that slopes above 740m altitude are strongly over-represented. Since these slopes are also gentler, the biased sampling may affect any of Nieuwenhuis and van den Berg's conclusions. Nevertheless, the square root transform (actually, where θ is the angle in degrees, $\sqrt{100 \tan \theta} + \sqrt{100 \tan \theta + 1}$) does provide a very linear probability plot.

A square root transformation was applied by Christofoletti and Tavares (1976), but to angles in degrees rather than tangents, i.e. they used $\sqrt{\theta} + \sqrt{\theta + 1}$. Aggregating to six classes, this gave a chi square value of 14.37 compared with 18.47 for a logarithmic transformation, 125.72 for no transformation of degrees, and 18.55 for the tabulated 99.5% confidence level for 6 degrees of freedom. Hence they concluded that the square root transformation gave a normal distribution. Stocking (1972) did not find it necessary to transform gradient (degrees), although he took the square root of a dependent variable (length of gullies) to minimise skewness. Schumm (1956) did not need to apply any transformation to his badland slopes; with means of 43 and 44 degrees, they were near-normal. There is no consensus then, on the transformation required to normalise gradient frequency distributions, or on whether a single transformation is widely applicable

METHODOLOGY FOR THE ASSESSMENT OF TRANSFORMATIONS Of the papers quoted above, only Speight (1971) and Christofoletti and Tavares (1976) made any serious attempt to compare different transformations in terms of their effect on fit to the normal frequency distribution model. Simply to show that one transformation reduces skewness, or produces a frequency distribution whose divergence from normal (for that sample size) is statistically insignificant, as have several other authors, is not conclusive in this context.

A chi square test is not of great value here, since it is sensitive to the number and limits of the classes used to compare observed and expected frequencies, and the classes usually used provide rather coarse nets. A Kolmogorov -Smirnov test is rather better since it permits the use of finer classes and is based on cumulated frequencies, taking ranking into account whereas chi square degrades a ratio scale of measurement to a nominal one. However, the fact remains that an insignificant deviation from normality in a small sample may be much more marked than a significant deviation from normality in a very large sample. Significance testing can be a red herring; it is more important in this context to take samples large enough to provide powerful comparisons between transformations, and to assess the degree and the character of deviation from normality. This viewpoint is strengthened by (i) the fact that slope profiles have usually been selected subjectively, or in some way that provides neither a random nor a systematic sample of the study area, hence preventing the application of statistical inference from the set of profiles to the area as a whole, and (ii) the autocorrelation of gradients along profiles or across matrices makes it very difficult to establish how many degrees of freedom are present; thinning out the data, i.e. discarding most of it, as do Nieuwenhuis and van den Berg, is hardly an ideal solution

To compare the degree of deviation from normality, skewness is without doubt the most important single statistic (followed by kurtosis). Values in the tail of a skewed distribution may greatly affect descriptive statistics and correlations, whereas those in the two tails of a leptokurtic distribution may often balance each other. It is desirable, then, to find a transformation which minimises skewness (Evans, Catterall and Rhind 1975). Given low skewness, normal kurtosis is the next desideratum.

A fuller picture of deviation from normality is provided by a plot on cumulative probability paper. Class limits are plotted on one axis against the cumulated percent frequency at those limits on the other axis: in the present paper frequencies are cumulated upward. Classes should be as small as possible, especially in the tails. Divisions on the paper are drawn so that normal frequency distributions plot as straight lines. Although the two tails of such a plot are important, we should beware of exaggerating the importance of a few extreme points, emphasised by the probability paper which 'stretches' both tails. With a horizontal cumulated frequency axis, skewed distributions plot concave (positive) or convex (negative) upward. Unskewed kurtic distributions plot S-shaped, balanced at the mean, with the central part steeper (platykurtic: broad mode or truncated tails) or flatter (leptokurtic: peaked mode or extended tails). More complex deviations from normality are reflected in other curves or breaks in the slope of the probability plot. It should not be assumed, however, that a break in slope on this plot marks the correct point for subdivision into two 'normal' components, for such supposed components must be replotted individually and may then be affected quite differently by the 'normal probability' transformation. This graphic technique is both robust and discriminating, and chief reliance is placed upon it here; the measurement of skewness is a suitable gross test, but skewness can be produced in different ways.

DATA(i) : ALTITUDE MATRICES. Large data sets are required to discriminate between different frequency distribution models of the types discussed by Speight (1971). Tables 1 and 2 give data in 1° classes, for sets of 3,447 to 11,582 measurements of gradient, while Figs 3 to 9 give the corresponding histograms Each is based on an altitude matrix, and meshes vary from 7.62m to 100m. These gradients are calculated not by the finite difference method used in Report 3, but by an improved method. This is implemented by the main terrain analysis program, discussed in detail in further reports in this series A local quadratic trend surface is fitted to each 3 x 3 submatrix, and the gradient at the centre of the submatrix is calculated by substitution into the trend surface equation Frequency distributions are tabulated for gradient and for the other derivatives of the altitude surface; aspect, profile convexity and plan convexity, as well as for altitude itself Since aspect is indeterminate when gradient is zero, such points are excluded from these tabulations.

Table 3 gives the moment measures of these gradient frequency distributions. Skewness is greatest for the two matrices (CACHE 1 and CACHE 2) with the lowest mean values, and it is lowest for the steep NUPUR and FERRO areas. TORRIDON, with a skewness of ·1 despite a high mean, is the exception. No cases of negative skewness occur. Kurtosis, as usual, increases with skewness. Two of the matrices with low skewness, CACHE 3 and NUPUR, have negative kurtosis (they are platykurtic, with truncated tails and/or broad modes relative to their standard deviations). FERRO, on the other hand, is leptokurtic despite a near-absence of skew. Hence despite the prevailing positive skewness, the seven gradient distributions do not obviously belong to the same family of frequency distributions This is confirmed by plots on probability paper

The Cache area (4 x 12 km, divided into 3 squares) is in Oklahoma, and extends from a lowland (CACHE 1) to an upland (CACHE 3), area with CACHE 2 a mixture of both. The data were produced by automatic photogrammetric profiling on a UNAMACE machine, followed by processing to remove noise (this involved a certain amount of smoothing) For the untransformed distributions (Figs 3, 4 and 5) skewness decreases with increasing gradient This is confirmed by the probability plots (Fig 10) which are very concave-up for CACHE 1, but straight above 2° (20%) for CACHE 3: CACHE 2 has an unusually steep plot.

The logtangent transformation (Fig.12) leaves some positive skew for CACHE 1, but overtransforms CACHE 3 for which the probability plot is dominantly convex-up, with some platykurtosis. CACHE 2 is more complex, producing an S-shaped curve with a steep central section from 40 to 90%; this is interpreted as heterogeneity, with a large gentle area comparable to CACHE 1, and a small area steeper than CACHE 3. The square root of sine plots are similar except that the concavity of CACHE 1 and the initial concavity of the other plots are more marked. Hence the logtangent transformation is preferred for Cache, although it is far from ideal.

The fine-meshed Gold Creek matrix describes a small drainage basin near Canberra, New South Wales, Australia. The considerable positive skew of its gradients requires transformation, and the logtangent transform seems appropriate despite minor bumps in the probability plot (Fig.11) : the square root of sine (Fig.15) leaves a slight positive skew.

The FERRO area of N.E. Calabria, Italy is also a drainage basin, but with slopes much steeper than Gold Creek. The 100m grid is of altitudes read from a photogrammetric 1/25,000 map, to the nearest 10m (i.e. one contour interval). Its gradients have the lowest skew of those from matrices, and are approximately normally distributed without transformation (Fig.13). The square root of sine transformation (Fig.15) is too drastic, producing a definite negative skew (upward convexity on the plot).

The NUPUR area is a glacially dissected plateau in northwest Iceland. Altitudes were read to the nearest 5m, on a 100m grid, from a 1/25,000 photogrammetric contour map. It is considerably steeper than the other areas, and its gradients are platykurtic but almost unskewed. Hence the square root of sine transformation (Fig.15) and the log tangent (Fig.14) exaggerate the convexity of the plot around 35° and produce negative skew. The TORRIDON area is a similar heavily glaciated mountain area but without plateau remnants. Altitudes were read to the nearest metre on a 100m grid, from the new Ordnance Survey 1/10,560 and 1/10,000 photogrammetric maps.

The untransformed distribution of gradients forms an S-shaped plot, steepest between 15° and 35° (63% and 94%) (Fig 13). Transformation reduces the initial concavity but exaggerates the later convexity, so little is to be gained

In summary, the logtangent transformation seems appropriate for the gentler areas, but does not normalise distributions fully. Steeper areas, especially the unglaciated FERRO basin, require no transformation.

DATA(ii) . GRADIENTS FOR VARIABLE TRIANGLES IN A MESH OF SURFACE SPECIFIC POINTS

Hormann (1968, 1971) has digitized a large number of contour maps by subjectively selecting significant surface points such as summits, passes and pits, with further points along significant lines such as ridges, channels and breaks in slope. Points are added until it is considered that a reasonable approximation to the land surface as mapped can be provided by linear interpolation between the points,whose (X,Y,Z) coordinates are digitised. For each point, all neighbouring points are recorded, and the surface is reconstructed by computer program as a mesh of triangular facets, the triangles being as equiangular as possible.

This type of digital terrain model is equally comprehensive but more concise than an altitude matrix, since the redundancy of information is minimised. It is, however, more subjective, and the varying area of the triangles means that they cannot carry equal weight. Hormann weights his frequency distributions by map area; all frequencies are expressed as percentages of total area. The merits of Hormann's system are discussed by Mark and Peucker (1975) and by Mark (1975b), and will be further considered in the Final Report on the present project

Table 4 lists five areas for which gradient histograms were published in Hormann (1971, p 54 and p 57),and one (Schiltach, Schwarzwald) for which a frequency distribution was given in Hormann (1968, p.141). Also given are the scales of source maps, ranging from 1/5,000 to 1/25,000, the total area,and the number of triangles used. Dividing area by number of triangles, then taking the square root, gives a weighted 'mean linear dimension' of the triangles;

this is roughly equivalent to the mesh of an altitude matrix. The Stallwang basin was digitised at two scales, permitting a comparison of results based on maps at 1/25,000 and 1/5,000. Data read from Hormann's table and histograms were corrected for closure errors of some 1%, so that they totalled exactly 100% for each area (Table 5).

Fig.17 shows that the logtangent overtransforms most distributions, producing negative skew (upward convexity on the probability plot). Only the plot for the Stallwang basin in the Bayerische Tertiarhügelland (Bavarian hill country of Tertiary rocks) is linear, and then only for the 1/25,000-based digitization : the more detailed work from the 1/5,000 map gives a broader spread of gradients and a negative skew. The Val Tuoi basin of the Silvretta Alps is near-linear, but is improved by the square root of sine transformation, with which it gives a linear plot from 0.5% to 99.5% cumulated frequency (Fig 18).

The Bayerische Tertiarhügelland gradients now have a slight concavity around 6° (30%) for 1/25,000, and around 9° (60%) for 1/5,000, followed by a broad convexity for the latter, so perhaps the root sine transformation is the best compromise between the two map scales. Gradients of the Schiltach basin in the Schwarzwald are now linear except for an aberration around 50°, well beyond the 99.5 percentile.

On the other hand gradients from part of the Ilz basin in the Bayerischer Wald (the Bavarian Forest, near the Czech and Austrian frontiers) are still negatively skewed, while those from the Kuchel basin of the North Calcareous Alps in Bavaria (unlike the Silvretta Alps) and the Mala Kaliao basin in Cameroun are strongly negatively skewed (over-transformed). These two data sets are much more nearly normal without transformation (Fig.16), but the Bayerischer Wald is then positively skewed and appears to need a different transformation, e.g. square root of tangent.

Hence it is difficult to generalise about these data sets from Hormann : the square root of sine is the best single transformation but at least two sets should not be transformed. The differences cannot be related to topography, since the two Alpine areas plot quite differently. Scale of source map, on the other hand, does produce differences in distribution shape.

DATA (iii) : RELIEF-BASED 1 KM AVERAGE GRADIENT FOR BOHEMIA AND MORAVIA

Kudrnovska (1972) produced an interesting data set for the whole of the Czech lands (52,475km^{2}). Range in altitude (relief, in metres) was calculated for each 1 x 1 km square from 1/25,000 maps, and multiplied by .001 to give the tangent of gradient. This was tabulated for five regions (Table 6) roughly equal in area, and (in the original) for many subdivisions. The use of eight classes make the data less detailed than the other sets used here, but the wisely chosen class limits 1,2,3,5,7,10 and 15 degrees provide as much information as possible, and permit use of the data for present purposes. Like methods (i) and (ii) the relief method samples the whole surface area systematically ; the averaging involved, however, means that we are dealing with gradient at a much coarser scale than with even the 100m grid mesh or 150m triangles.

Probability plots show that the central region is consistently gentlest, and northern and eastern regions have greatest slope dispersion, mixing the steepest slopes with a considerable number below 1°. Logtangent (Fig.19) and also logdegrees plots are all slightly convex-up (negatively skewed), very markedly so for the central region. Rootsine plots are slightly concave-up, except for the central region which is just on the convex side of straight (Fig.20). As in Speight's (1971) study, and despite the difference in scale, the 'ideal' transform is somewhere between the logarithm of tangent and the square root of sine : but in the Czech case the latter has the edge.

DATA (iv) SLOPE PROFILES, FIELD-SURVEYED

Many British geomorphologists are distrustful of data obtained from medium-scale maps (e.g. Pitty, 1969) and might maintain that gradients of slopes profiled in the field are of much greater interest than any of the above. The techniques and problems involved were discussed by Pitty (1969) and by Young (1972). Most recent work has been based on measurement over fixed increments of slope length; results differ according to the slope length selected (Gerrard and Robinson, 1971). But the main problem with the measurement of profiles from hillcrest to drainage line is the apparent impossibility of selecting a random or systematic sample of profiles (Young, 1972, p.145: Reynolds, 1975), compounded by difficulties of access or of anthropogenic modification for some of the profiles selected, which usually cause their disqualification. Hence it is usually necessary to regard a set of slope profiles as a subjective 'sample' of an area, or as a sample of certain types of slope (e.g. straight in plan) only.

The first such data are taken from the complete distributions of Nieuwenhuis and van den Berg (1971), divided for lithology (Table 7). Gradient was measured for 6,034 unit lengths of 10m on profiles subjectively located within part of the Morvan, with some bias toward higher altitudes (with gentler slopes). Both are overtransformed by logtangent (Fig.21), but quite normal as square root of sine (Fig.22).

Second, Juvigne (1973) measured some 40km of profiles in the Famenne region of Belgium; percentage frequencies of gradientover 200m unit lengths, read from his Fig.6b are given in Table 8. This distribution has an awkward tail of high gradients and requires severe transformation : even logtangent has a positive skew (Fig.21). N.J. Cox (unpublished) has provided data for 4,571 unit lengths of 1.5m on eleven profiles in the North Yorkshire Moors, England. These form a subjective sample of straight slopes undisturbed by for example roads or quarries, above headstreams in 10 x 10km grid square SE59. Gradient was measured to the nearest $\frac{1}{2}°$ with a slope pantometer. Table 9 gives frequencies and cumulative percentages after 51 zero and 113 negative gradients were discarded. The square root of sine transformation (Fig 22) does not fully remove the positive skew, but the logtangent (Fig.23) provides an almost normal distribution.

Another selective data set comes from Tinkler (1966), who surveyed 46 closely-spaced profiles on the Eglwyseg Carboniferous Limestone scarp slope, between Wrexham and Llalgollen, N.E. Wales (Table 10). Since the cliff above, and gentler slopes in the valley, were excluded by definition, the gradients have unusually low variability. Being in the range 16-44 degrees, they are little affected either by logtangent (Fig.23) or (Fig.24) rootsine transformation, but both probability plots are convex-up. Even the untransformed plot (Fig.16) is convex-up, showing negative skewness. Hence these data should not undergo any of the usual transformations. A very similar plot (on Fig.13) is provided by slopes from a quite different environment, the dissected Neogene Basin fill of central Afghanistan (Table 11: Evans, 1964). These gradients are almost normally distributed without transformation. The small negative skew in both cases probably relates to the existence of a maximum gradient on which a waste mantle can be maintained.

Gerrard and Robinson (1971) made an interesting comparison between measured lengths of 2.5, 5 and 10m on the same 30 randomly-located profiles in the New Forest, Hampshire (Table 12). All three distributions plot strongly convex-up on a logtangent transformation (Fig.23), but those for 2.5m measured lengths are nearly straight on the rootsine probability plot (Fig.24). Those for 5m and especially 10m are progressively more convex-up and require a weaker transformation to remove their small positive skew on the degrees scale.

Pitty (1970) calculated both moment - and percentile-based measures of skewness for individual slope profiles. His dissatisfaction (p.5) with moment measures due to the considerable effect of outliers can be related in part to (i) the small number of measurements per profile - it is desirable to combine many profiles before calculating moment measures; (ii) the short unit length of 1.52m; (iii) a technique of profiling along straight lines, whereby local reversals produce 'negative gradients'; and (iv) the exclusion of large parts of the land surface. He found a broad range of both positive and negative skewness, the latter being much more likely for profiles with median gradients in excess of 20°.

Although there is some regional consistency, the diversity of types of skewness suggests the need for various types of transformation, as Pitty concluded on p.12.

Field-measured slope-profile gradient data, then, are as diverse as the other types. Some require rootsine transformation, some logtangent, and some no transformation at all. Frequency distributions vary with scale (length of unit measurements), with type of region and probably with technique.

Effect of horizontal matrix resolution on shape of frequency distributions

Returning to gradient calculated from altitude matrices, it is possible to recalculate these for 'thinned' matrices, as if the matrix had coarser resolution, by using only every nth point. This was done for Report 3, where the effect on mean and standard deviation of gradient was considered at length. Table 13 gives full moment-based descriptive statistics for some 'thinned' versions of Torridon and Cache 2. It shows that skewness and kurtosis are less sensitive to mesh than are mean and standard deviation. For extreme thinning, few points are involved and results are erratic, but there is no consistent tendency for skewness and kurtosis to increase or decrease with mesh.

Table 14 shows skewness as a function of resolution for four altitude matrices. These results were produced by a different program, which considers all possible thinned matrices instead of just one centrally-located thinned matrix : this gives a much larger set of measurements for larger values of n, since points are lost only around the edge. There is a tendency for skewness to decline very slowly as resolution is reduced (n is increased), but this is sometimes reversed.

It seems that the essential characteristics of the shape of a gradient frequency distribution are not greatly changed by changing resolution : differences between areas remain, with Cache 3 the least skewed of these four and Cache 1 and 2 the most skewed. Inspection of corresponding histograms confirms this constancy of character; for example, the bimodality and positive skew of Cache 2 persists even with extreme thinning.

Conclusions

In this analysis, a number of large data sets generated in four different ways have been plotted in comparable fashion. Where possible, the effect of scale (resolution) has been assessed. Regardless of data type, it is found that no one transformation permits normality to be achieved Positive skew is most widespread, but some data sets are (slightly) negatively skewed. For those which are near-normal without transformation, it seems undesirable to split them up into logtangent-normal components, as did Speight (1971). On the other hand, some data sets such as Cache 2 are obviously compound and might best be subdivided.

The logarithm of tangent and the square root of sine are the most widely useful transformations, but it is necessary to maintain an open mind and try different transformations for some data sets. As yet it is difficult to speculate on relations between the frequency distribution of gradients in a particular area, and the processes and modes of slope development operating.

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Table [illegible]. Frequency distributions of gradient from altitude matrices ul = (exclusive) upper limit of class in degrees, f = frequency, c = cumulated percent frequency

	CACHE 1		CACHE 2		CACHE 3		GOLD CR.	
u l	f	c	f	c	f	c	f	c
0.5	245	2.87	132	1.47	48	.51	4	.12
1.5	4719	58.13	3662	42.24	1071	11.80	117	3.51
2.5	2427	86.55	2113	65.76	1287	25.38	369	14.22
3.5	772	95.59	703	73.58	846	34.30	629	32.46
4.5	229	98.27	372	77.72	692	41.60	624	50.57
5.5	90	99.32	229	80.27	789	49.92	571	67.13
6.5	26	99.63	172	82.19	780	58.15	389	78.42
7.5	15	99.80	170	84.08	703	65.56	264	86.07
8.5	6	99.87	176	86.04	782	73.81	169	90.98
9.5	4	99.92	205	88.32	720	81.40	108	94.11
10.5	0	99.92	238	90.97	624	87.99	61	95.88
11.5	2	99.94	205	93.25	466	92.90	41	97.07
12.5	1	99.95	190	95.37	319	96.27	35	98.08
13.5	1	99.96	122	96.73	160	97.95	26	98.84
14.5	2	99.99	89	97.72	82	98.82	5	98.98
15.5	1	100.00	87	98.69	49	99.34	13	99.36
16.5			47	99.21	30	99.65	10	99.65
17.5			27	99.51	14	99.80	3	99.74
18.5			18	99.71	7	99.87	6	99.91
19.5			11	99.83	7	99.95	2	99.97
20.5			7	99.91	0	99.95	1	100.00
21.5			2	99.94	3	99.98		
22.5			3	99.97	2	100.00		
23.5			3	100.00				
24.5								
25.5								

Table 2 Frequency distributions of gradient from altitude matrices. ul = (exclusive) upper limit of class in degrees, f = frequency, c = cumulated percent. frequency.

FERRO		NUPUR		TORRIDON		
f	c	f	c	f	c	ul
0	.00	82	1.35	0	.00	0.5
124	1.07	59	2.32	218	2.33	1.5
139	2.27	119	4.27	370	6.27	2.5
261	4.52	136	6.51	369	10.21	3.5
173	6.02	133	8.69	495	15.49	4.5
209	7.82	171	11.51	493	20.75	5.5
287	10.30	151	13.99	454	25.60	6.5
289	12.80	157	16.57	525	31.18	7.5
425	16.47	135	18.79	511	36.63	8.5
584	21.51	142	21.12	488	41.84	9.5
687	27.44	146	23.52	497	47.14	10.5
1105	36.98	139	25.81	402	51.43	11.5
929	45.00	159	28.42	343	55.09	12.5
1100	54.50	166	31.15	315	58.45	13.5
870	62.01	167	33.89	274	61.37	14.5
928	70.02	179	36.83	268	64.23	15.5
716	76.20	183	39.84	216	66.54	16.5
663	81.93	145	42.23	220	68.89	17.5
559	86.76	132	44.40	198	71.00	18.5
404	90.24	136	46.63	172	72.83	19.5
325	93.05	142	48.96	152	74.46	20.5
254	95.24	115	50.85	151	76.07	21.5
194	96.92	146	53.25	138	77.54	22.5
114	97.90	138	55.52	133	78.96	23.5
88	98.66	157	58.10	136	80.41	24.5
50	99.09	145	60.49	120	81.69	25.5
35	99.40	151	62.97	108	82.84	26.5
29	99.65	132	65.14	103	83.94	27.5
10	99.73	136	67.37	102	85.03	28.5
15	99.86	146	69.77	114	86.25	29.5
7	99.92	127	71.86	96	87.27	30.5
0	99.92	132	74.03	93	88.26	31.5
3	99.95	136	76.27	110	89.44	32.5
2	99.97	139	78.55	99	90.49	33.5
2	99.98	138	80.82	95	91.51	34.5
2	100.00	145	83.20	96	92.53	35.5
		133	85.39	108	93.68	36.5
		120	87.36	99	94.74	37.5
		123	89.38	92	95.72	38.5
		104	91.09	64	96.40	39.5
		109	92.88	84	97.30	40.5
		73	94.08	64	97.98	41.5
		80	95.40	55	98.57	42.5
		63	96.43	31	98.90	43.5
		63	97.47	29	99.21	44.5
		44	98.19	17	99.39	45.5
		23	98.57	7	99.47	46.5
		22	98.93	8	99.55	47.5
		16	99.19	16	99.72	48.5
		11	99.38	3	99.75	49.5
		11	99.56	4	99.80	50.5
		6	99.65	9	99.89	51.5
		7	99.77	2	99.91	52.5
		5	99.85	5	99.97	53.5
		3	99.90	2	99.99	54.5
		2	99.93	1	100.00	55.5
		1	99.95			56.5
		1	99.97			57.5
		1	99.98			58.5
		1	100.00			59.5
						60.5

Table 3. Moment measures for gradients from altitude matrices. Gradients are calculated by local quadratic method and exclude zero values: figures in brackets are based on finite difference method and include zero gradients.

	No. of points	Mean	St. Dev.	Skewness	Kurtosis	Grid mesh (m)
CACHE 1	8,540	1.563	1.008	2.921	19.46	25
CACHE 2	8,983 (9,604)	3.450 (3.16)	3.877 (3.86)	1.829 (1.93)	2.58 (.234)	25
CACHE 3	9,481	5.844	3.695	.458	- .52	25
GOLD CR.	3,447	4.950	2.649	1.447	3.36	7.62
FERRO	11,582	13.087	5.086	.144	.75	100
NUPUR	6,084	21.627	12.586	.176	-.97	100
TORR	9,372 (9,604)	14.761 (14.93)	11.090 (12.49)	1.009 (1.125)	.149 (-2.12)	100

Table 4. Characteristics of data sets derived from Hormann (1968, 1971).

Code	Region	Drainage basin	Map scale	Area (km^2)	No. of triangles	Mean linear dimension
233	945/7/11	Stallwang	1/5,000	3.302	1963	41m
	Landshut, Isar Basin, Bayerische Tertiarhügelland					
233	945/5/11	"	1/25,000	3.253	142	151m
	"	"				
253	8133/6/10	Kuchel	1/10,000	10.513	2531	64m
	Elmau (part of Linder-Ammer) basin, Northern Calcareous Alps, Bavaria.					
239	588/7/11	Ilz (part of)	1/5,000	1.584	1426	33m
	Bayerischer Wald, eastern part.					
242	27/5/3	Val Tuoi	1/25,000	26.452	1050	159m
	Silvretta Group, Unter-Engadin, Swiss Alps					
13521/5/4		Schiltach, above Lauterbach	1/25,000	55.395	929	244m
		Kinzig basin, northern Schwarzwald.				
777	3401/5/9	Mayo Kaliao	1/25,000 (enlarged from 1/50,000)	60.277	2498	155m
	Part of Mayo Débi basin, Tsanaga-Logone basin, N.E. Cameroun (14°4 E, 10°40 N)					

Table 5 Frequency distributions of gradient from Hormann (1968,1971)

ul = upper limit of class (in degrees)
p = percent frequency, c = cumulated percent frequency

	Kuchel,N Calc.Alps		Val Tuoi Silvretta		Ilz,Bayer Wald	
u.l.	p	c	p	c	p	c
2	.1	.1	.0	.0	3.0	3.0
4	.2	.3	.4	.4	7.2	10.2
6	.3	.6	.7	1.1	6.6	16.8
8	.1	.7	1.2	2.3	9.5	26.3
10	.2	.9	1.5	3.8	9.3	35.6
12	.3	1.2	3.0	6.8	9.0	44.6
14	.1	1.3	3.8	10.6	9.7	54.3
16	.5	1.8	4.6	15.2	9.0	63.3
18	.8	2.6	4.3	19.5	7.7	71.0
20	.9	3.5	5.8	25.3	8.0	79.0
22	1.5	5.0	6.5	31.8	6.1	85.1
24	2.1	7.1	9.3	41.1	5.8	90.9
26	1.9	9.0	7.4	48.5	4.2	95.1
28	3.4	12.4	6.6	55.1	2.5	97.6
30	5.4	17.8	6.2	61.3	1.3	98.9
32	5.6	23.4	5.8	67.1	.5	99.4
34	9.6	33.0	5.6	72.7	.3	99.7
36	11.1	44.1	3.8	76.5	.1	99.8
38	12.3	56.4	2.3	78.8	.05	99.85
40	10.5	66.9	3.2	82.0	.05	99.9
42	7.6	74.5	4.6	86.6	.05	99.95
44	6.0	80.5	2.9	89.5	.05	100.0
46	4.3	84.8	2.3	91.8		
48	4.2	89.0	1.3	93.1		
50	2.8	91.8	1.8	94.9		
52	1.9	93.7	.9	95.8		
54	2.2	95.9	1.5	97.3		
56	1.2	97.1	.4	97.7		
58	1.1	98.2	.5	98.2		
60	.6	98.8	.1	98.3		
62	.7	99.5	.4	98.7		
64	.3	99.8	.3	99.0		
66	.2	100.0	.3	99.3		
68			.3	99.6		
70			.2	99.8		
			.2	100.0		

	Schiltach, Schwarzwald		Mayo Kaliao Cameroun		Stallwang, Bayer. Tertiarhügelland 1/5,000		1/25,000	
u.l.	p	c	p	c	p	c	p	c
1			4.5	4.5	.3	.3	.0	.0
2	2.23	2.23	16.2	20.7	2.2	2.5	.0	.0
3	4.72	6.95	17.9	38.6	3.4	5.9	.7	.7
4	4.89	11.84	8.8	47.4	6.1	12.0	2.6	3.3
5	3.60	15.44	5.5	52.9	8.7	20.7	10.4	13.7
6	4.03	19.47	3.5	56.4	11.4	32.1	17.0	30.7
7	6.32	25.79	3.0	59.4	11.3	43.4	16.7	47.4
8	5.51	31.30	2.3	61.7	8.4	51.8	9.4	56.8
9	8.34	39.64	2.2	63.9	9.5	61.3	9.8	66.6
10	6.22	45.86	1.5	65.4	6.0	67.3	7.3	73.9
11	6.62	52.48	1.5	66.9	3.6	70.9	8.0	81.9
12	6.89	59.37	2.1	69.0	4.5	75.4	3.5	85.4
13	5.47	64.84	1.7	70.7	3.2	78.6	3.0	88.4
14	6.00	70.84	2.0	72.7	2.8	81.4	3.7	92.1
15	4.48	75.32	2.7	75.4	3.2	84.6	3.3	95.4
16	4.78	80.10	1.6	77.0	2.0	86.6	.9	96.3
17	2.98	83.08	1.7	78.7	1.9	88.5	1.1	97.4
18	3.41	86.49	1.6	80.3	2.0	90.5	1.7	99.1
19	2.00	88.49	2.4	82.7	1.7	92.2	.0	99.1
20	2.69	91.18	1.7	84.4	1.3	93.5	.3	99.4
21	2.19	93.37	1.6	86.0	1.8	95.3	.0	99.4
22	.96	94.33	1.7	87.7	1.2	96.5	.25	99.65
23	.83	95.16	1.8	89.5	.9	97.4	.25	99.9
24	.85	96.01	1.6	91.1	.6	98.0	.0	99.9
25	.51	96.52	2.1	93.2	.5	98.5	.0	99.9
26	.24	96.76	1.0	94.2	.4	98.9	.1	100.0
27	.86	97.62	1.3	95.5	.6	99.5		
28	.44	98.06	1.2	96.7	.3	99.8		
29	.50	98.56	.9	97.6				
30	.33	98.89	.8	98.4	.1	99.9		
31	.12	99.01	.4	98.8				
32	.00	99.01	.4	99.2				
34	.36	99.37	.6	99.8	.1	100.0		
36	.08	99.45	.2	100.0				
38	.19	99.64						
44	.02	99.66						
46	.11	99.77						
48	.03	99.80						
50	.02	99.82						
54	.15	99.95						
56	.03	99.98						
58	.01	99.99						
66	.01	10.00						

Table 6. Frequency distributions of average gradient per 1x1 km square in the Czech Lands (Bohemia and Moravia), from Kudrnovska (1972). Note that tangent (average gradient) was calculated by multiplying range in altitude by .001: this involves assumptions about the separation of the highest and lowest points in each square, and the appropriate quotient might vary between e.g. .0008 and .0012 in different types of topography.

ul = upper class limit (degrees),

p = percentage frequency, c = cumulated percent frequency. Note the varying class width.

REGION AREA(km^2)	CENTRAL 11,209		SOUTHERN 11,344		WESTERN 10,872		NORTHERN 7,810		EASTERN 11,240	
ul	p	c	p	c	p	c	p	c	p	c
1	13.81	13.81	8.23	8.23	1.53	1.53	4.35	4.35	10.65	10.65
2	18.16	31.97	17.46	25.69	10.46	11.99	11.73	16.08	17.75	28.40
3	18.84	50.81	19.20	44.89	17.16	29.15	14.19	30.27	16.71	45.11
5	27.92	78.73	29.13	74.02	32.37	61.52	25.72	55.99	25.04	70.15
7	13.55	92.28	13.33	87.35	18.21	79.73	16.16	72.15	13.14	83.29
10	6.64	98.92	9.05	96.40	13.12	92.85	15.12	87.27	9.88	93.17
15	1.07	99.99	3.34	99.74	6.10	98.95	10.31	97.58	5.04	98.21
(>15)	.01	100.00	0.26	100.00	1.05	100.00	2.42	100.00	1.79	100.00

Table 7. Frequency distributions of gradient over 10m unit lengths on subjectively located profiles in Morvan, Central France, read off Figs. 5 & 6 in Nieuwenhuis and van den Berg (1971). Tangent θ was originally measured to the nearest 1%, then grouped into tangent classes of 3% below .21 and 6% above.

ul = upper limit of class, p = percentage frequency, c = cumulated percent frequency

ul tan θ	ul degrees	ul $\sqrt{sine}$	ul logtan	2925 lengths on tuff p	c	3109 lengths on microgranite p	c
.025	1.43	.158	-1.602	9.0	9.0	16.3	16.3
.055	3.15	.234	-1.260	13.5	22.5	16.8	33.1
.085	4.86	.291	-1.071	12.4	34.9	12.0	45.1
.115	6.56	.338	- .939	9.9	44.8	10.7	55.8
.145	8.25	.379	- .839	10.9	55.7	8.9	64.7
.175	9.93	.415	- .757	8.9	64.6	6.2	70.9
.205	11.59	.448	- .688	8.9	73.5	5.6	76.5
.265	14.84	.506	- .577	11.9	85.4	9.1	85.6
.325	18.00	.556	- .488	7.5	92.9	6.6	92.2
.385	21.06	.599	- .415	3.8	96.7	3.8	96.0
.445	23.99	.638	- .352	1.2	97.9	1.8	97.8
.505	26.79	.671	- .297	1.0	98.9	0.8	98.6
.625	32.01	.728	- .204	0.8	99.7	1.2	99.8
(>.625)				0.3	100.0	.2	100.0

Table 8. Frequency distributions of gradient over 200m unit lengths in Famenne, southeast Belgium, read off Fig.6b in Juvigne (1973). ul = upper limit of class (in degrees), p = percent frequency, c = cumulated percent frequency

ul	p	c	ul	p	c
0.5	3.3	3.3	15.5	.1	97.4
1.0	9.0	12.3	16.0	.1	97.5
1.5	5.4	17.7	16.5	.1	97.6
2.0	9.6	27.3	17.0	0	97.6
2.5	7.1	34.4	17.5	.1	97.7
3.0	10.7	45.1	20.5	.1	97.8
3.5	7.5	52.6	21.0	.1	97.9
4.0	8.3	60.9	21.5	.1	98.0
4.5	4.2	65.1	23.0	.1	98.1
5.0	6.2	71.3	23.5	0	98.1
5.5	2.8	74.1	24.0	.1	98.2
6.0	5.2	79.3	24.5	.1	98.3
6.5	2.9	82.2	25.0	0	98.3
7.0	3.0	85.2	25.5	.3	98.6
7.5	3.1	88.3	26.0	.1	98.7
8.0	1.6	89.9	26.5	.1	98.8
8.5	.5	90.4	27.0	0	98.8
9.0	1.9	92.3	27.5	.1	98.9
9.5	1.2	93.5	28.0	0	98.9
10.0	.7	94.2	28.5	.1	99.0
10.5	.7	94.9	29.0	0	99.0
11.0	.4	95.3	29.5	.1	99.1
11.5	.7	96.0	31.5	.1	99.2
12.0	.3	96.3	32.0	0	99.2
12.5	.4	96.7	32.5	.1	99.3
13.0	.3	97.0	33.0	0	99.3
13.5	.1	97.1	33.5	.2	99.5
14.0	.1	97.2	34.0	0	99.5
14.5	.1	97.3	34.5	.1	99.6
15.0	0	97.3	>37	.4	100.0

Table 9. Frequency distribution of gradient over 1.5m unit lengths on profiles of 11 straight slopes undisturbed by e.g. roads, above headstreams in 10 x 10km grid square SE59, North Yorkshire Moors. Unpublished field measurements to the nearest 0.5° with a slope pantometer, kindly provided by N.J. Cox.
ul = upper limit of class, f = number of unit lengths, c = cumulated percent. frequency. Ungrouped below 10°, aggregated into 2° classes from 10° upward.

ul	f	c	ul	f	c
.75	37	.84	11.75	460	69.75
1.25	64	2.29	13.75	324	77.10
1.75	46	3.34	15.75	231	82.35
2.25	86	5.29	17.75	159	85.95
2.75	94	7.42	19.75	129	88.88
3.25	133	10.44	21.75	88	90.88
3.75	127	13.32	23.75	80	92.69
4.25	151	16.75	25.75	53	93.90
4.75	174	20.69	27.75	47	94.96
5.25	236	26.05	29.75	40	95.87
5.75	149	29.43	31.75	52	97.05
6.25	220	34.42	33.75	28	97.69
6.75	171	38.30	35.75	19	98.12
7.25	209	43.05	37.75	14	98.43
7.75	173	46.97	39.75	17	98.82
8.25	166	50.74	41.75	10	99.05
8.75	147	54.07	43.75	8	99.23
9.25	133	57.09	45.75	11	99.48
9.75	98	59.31	47.75	3	99.55
			49.75	0	99.55
			51.75	15	99.89
			65.75	4	99.98
			90.00	1	100.00

Table 10. Frequency distribution of gradients measured by Abney level to the nearest degree along 46 profiles on the Eglwyseg scarp, Clwyd, N.E. Wales, from Fig 4 in Tinkler (1966).

ul = upper limit of class, in degrees. f = number of slope facets (of varying length : average 7.62m)

c = cumulated percentage frequency

ul	f	c	ul	f	c
16.5	1	.25	31.5	24	35.61
17.5	2	.76	32.5	32	43.69
18.5	1	1.01	33.5	33	52.02
19.5	1	1.26	34.5	47	63.89
20.5	0	1.26	35.5	60	79.04
21.5	8	3.28	36.5	26	85.61
22.5	2	3.79	37.5	14	89.14
23.5	6	5.30	38.5	14	92.68
24.5	6	6.82	39.5	8	94.70
25.5	13	10.10	40.5	13	97.98
26.5	15	13.89	41.5	7	99.75
27.5	10	16.41	42.5	0	99.75
28.5	9	18.69	43.5	0	99.75
29.5	20	23.74	44.5	1	100.00
30.5	23	29.55			

Table 11. Frequency distribution of maximum hillside gradient on Neogene basin fill between Bamian and the Koh-i-Baba in Central Afghanistan. Measured by Evans (1964, Fig.5.02) to the nearest degree, by Abney level, at subjectively located points.

ul	f	c	ul	f	c
16.5	1	.37	31.5	20	44.69
17.5	0	.37	32.5	11	48.72
18.5	1	.73	33.5	23	57.14
19.5	2	1.47	34.5	35	69.96
20.5	1	1.83	35.5	20	77.29
21.5	4	3.30	36.5	15	82.78
22.5	0	3.30	37.5	19	89.74
23.5	4	4.76	38.5	8	92.67
24.5	11	8.79	39.5	4	94.14
25.5	12	13.19	40.5	7	96.70
26.5	15	18.68	41.5	3	97.80
27.5	10	22.34	42.5	1	98.17
28.5	8	25.27	43.5	3	99.27
29.5	17	31.50	44.5	2	100.00
30.5	16	37.36			

Table 12. Frequency distributions of gradient for 30 randomly selected slopes in the New Forest, Hampshire, England, read from Fig.1 of Gerrard and Robinson (1971). The same slopes were measured three times, with unit lengths of 2.5m, 5m and 10m. The mean varies only from 9.0 to 9.2 degrees, but the maximum varies from 22 to 30 degrees.

ul = class upper limit, f = number of unit lengths, c = cumulative percentage of unit lengths

	2.5 m		5 m		10 m	
ul	f	c	f	c	f	c
0.5	16	2.50	10	3.12	5	3.03
1.5	23	6.10	10	6.23	4	5.45
2.5	30	10.80	14	10.59	3	7.27
3.5	33	15.96	15	15.26	11	13.94
4.5	32	20.97	20	21.50	9	19.39
5.5	47	28.33	17	26.79	16	29.09
6.5	48	35.84	31	36.45	14	37.58
7.5	46	43.04	25	44.24	10	43.64
8.5	47	50.39	27	52.65	12	50.91
9.5	38	56.34	15	57.32	7	55.15
10.5	39	62.44	23	64.49	10	61.21
11.5	31	67.29	11	67.91	11	67.88
12.5	34	72.61	16	72.90	12	75.15
13.5	37	78.40	9	75.70	6	78.79
14.5	15	80.75	13	79.75	5	81.82
15.5	25	84.66	19	85.67	9	87.27
16.5	21	87.95	13	89.72	9	92.73
17.5	10	91.08	9	92.52	6	96.36
18.5	19	94.05	7	94.70	2	97.58
19.5	9	95.46	5	96.26	1	98.18
20.5	4	96.09	4	97.51	1	98.79
21.5	12	97.97	2	98.13	1	99.39
22.5	4	98.59	2	98.75	1	100.00
23.5	4	99.22	3	99.69		
24.5	1	99.37	0	99.69		
25.5	2	99.69	0	99.69		
26.5	0	99.69	0	99.69		
27.5	1	99.84	1	100.00		
28.5	0	99.84				
29.5	0	99.84				
30.5	1	100.00				

Table 13. Effect of grid mesh on moments of gradient frequency distributions. Zero gradients are included. Note that for n = 15 and more, the number of gradients measured is too small for reliable estimation of moments. *The multiple results for a single thinning represent differently positioned thinned matrices : this gives a rough idea of the stability of these results.

			ARITHMETIC (degrees)					LOGARITHMIC			
THINNING n	GRID MESH, m	NO. OF GRADIENTS	MAXIMUM	MEAN	ST.DEV.	SKEW	KURT.	MEAN	ST.DEV.	SKEW.	KURT.
		TORRIDON									
1	100	9604	73.93	14.93	12.49	1.125	-2.12	1.04	.422	-.788	-2.62
2	200	2304	51.40	14.15	10.66	.908	-3.08	1.06	.351	-.567	-2.84
3	300	930	40.00	12.88	9.10	.812	-3.27	1.04	.322	-.486	-3.10
5	500	324	31.40	10.78	6.68	.682	-3.28	.993	.275	-.460	-3.33
10	1000	64	12.64	5.96	2.77	.111	-3.85	.803	.198	-.688	-3.15
15	1500	16	6.73	3.79	1.78	-9.850	-3.96	.649	.177	-.500	-3.74
	CACHE 2										
1	25	9604	23.94	3.16	3.86	1.93	.23	.481	.330	.507	-3.25
2	50	2304	* 19.20	2.84	3.87	1.81	-.65	.425	.349	.728	-3.41
			19.20	3.11	3.74	1.85	-.49	.488	.304	.920	-3.11
			19.20	3.01	3.72	1.91	.24	.474	.307	.896	-3.05
3	75	930	* 17.03	2.94	3.55	1.78	-.90	.473	.299	.997	-3.15
			17.03	2.94	3.57	1.84	-.67	.475	.295	1.05	-3.03
5	125	324	12.83	2.62	3.07	1.71	-1.19	.448	.285	1.02	-3.21
10	250	64	8.44	2.03	2.21	1.56	-1.66	.396	.257	.93	-3.41
15	325	16	6.64	1.68	1.95	1.79	-.86	.352	.244	1.26	-2.44
16	350	16	5.76	1.49	1.56	1.93	.15	.338	.214	1.25	-2.15
20	500	9	5.90	1.64	1.78	2.03	1.38	.356	.235	1.18	-2.06

Table 14. Skewness of gradient as the matrix is thinned by taking every nth point in each direction. Zero gradients are included. The initial (n=1) grid mesh is 25m for Cache and 100m for Torridon.

n	TORRIDON	CACHE 1	CACHE 2	CACHE 3
1	1.00	2.03	1.86	.47
2	.94	1.54	1.85	.38
3	.86	1.40	1.82	.35
4	.76	1.28	1.79	.34
5	.67	1.21	1.76	.35
6	.61	1.17	1.72	.37
7	.57	1.11	1.66	.40
8	.51	1.05	1.60	.39
9	.56	1.00	1.55	.39
10	.63	.92	1.51	.34
11	.62	.86	1.46	.26
12	.75	.84	1.47	.13
13	.83	.83	1.43	.11
14	.69	.87	1.43	.03
15	.75	.88	1.40	.04
16	1.03	.94	1.46	.04
17	.65	.90	1.47	.02
18	.69	.78	1.45	.49
19	.79	.77	1.50	.27
20	.55	.90	1.48	-.16
21	.46	.81	1.20	-.09
22	.51	.77	1.20	.13
23	.55	.63	1.17	.24
24	.58	.54	1.21	.27
25	.61	.55	1.30	-.19

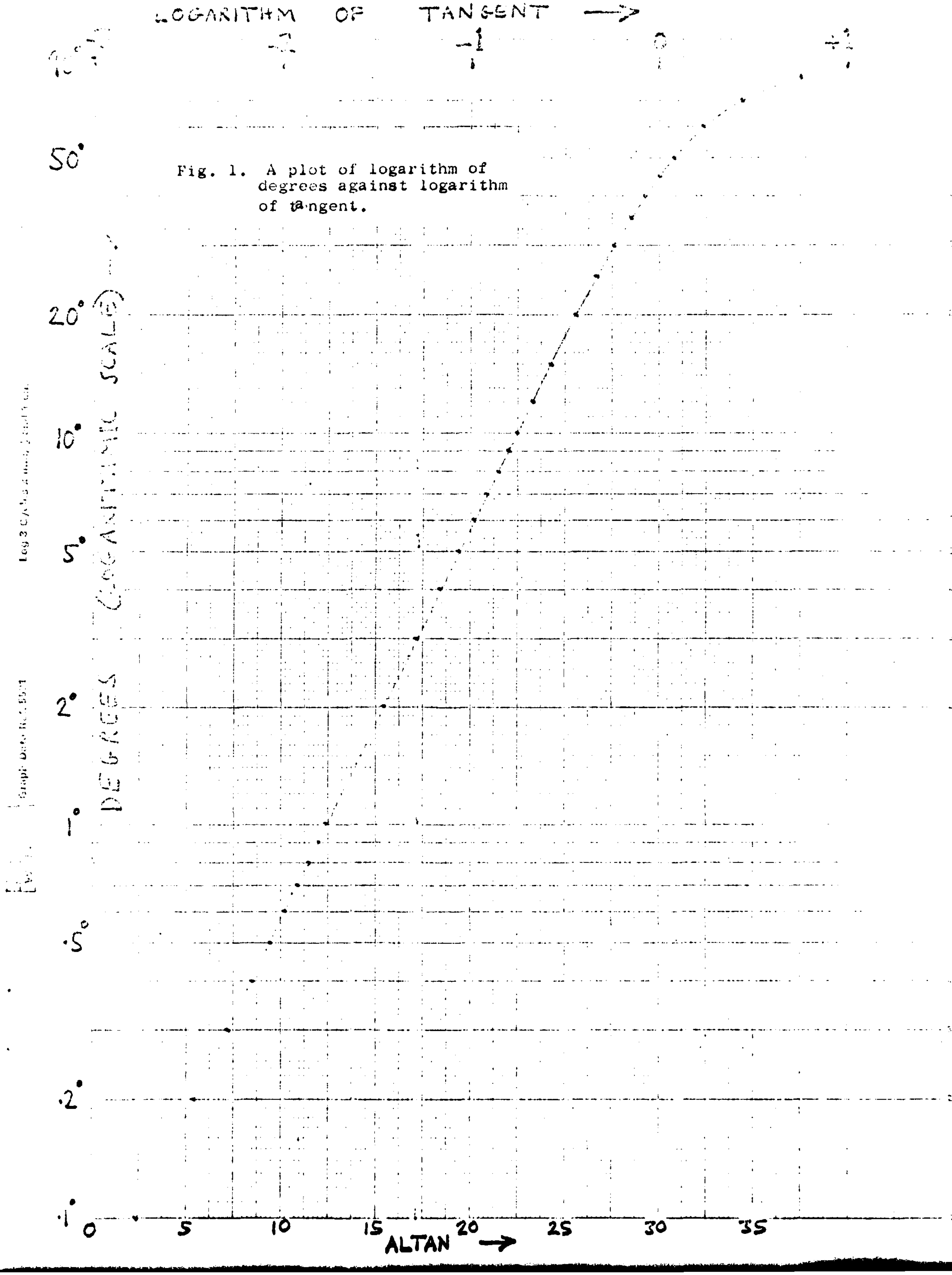

Fig. 1. A plot of logarithm of degrees against logarithm of tangent.

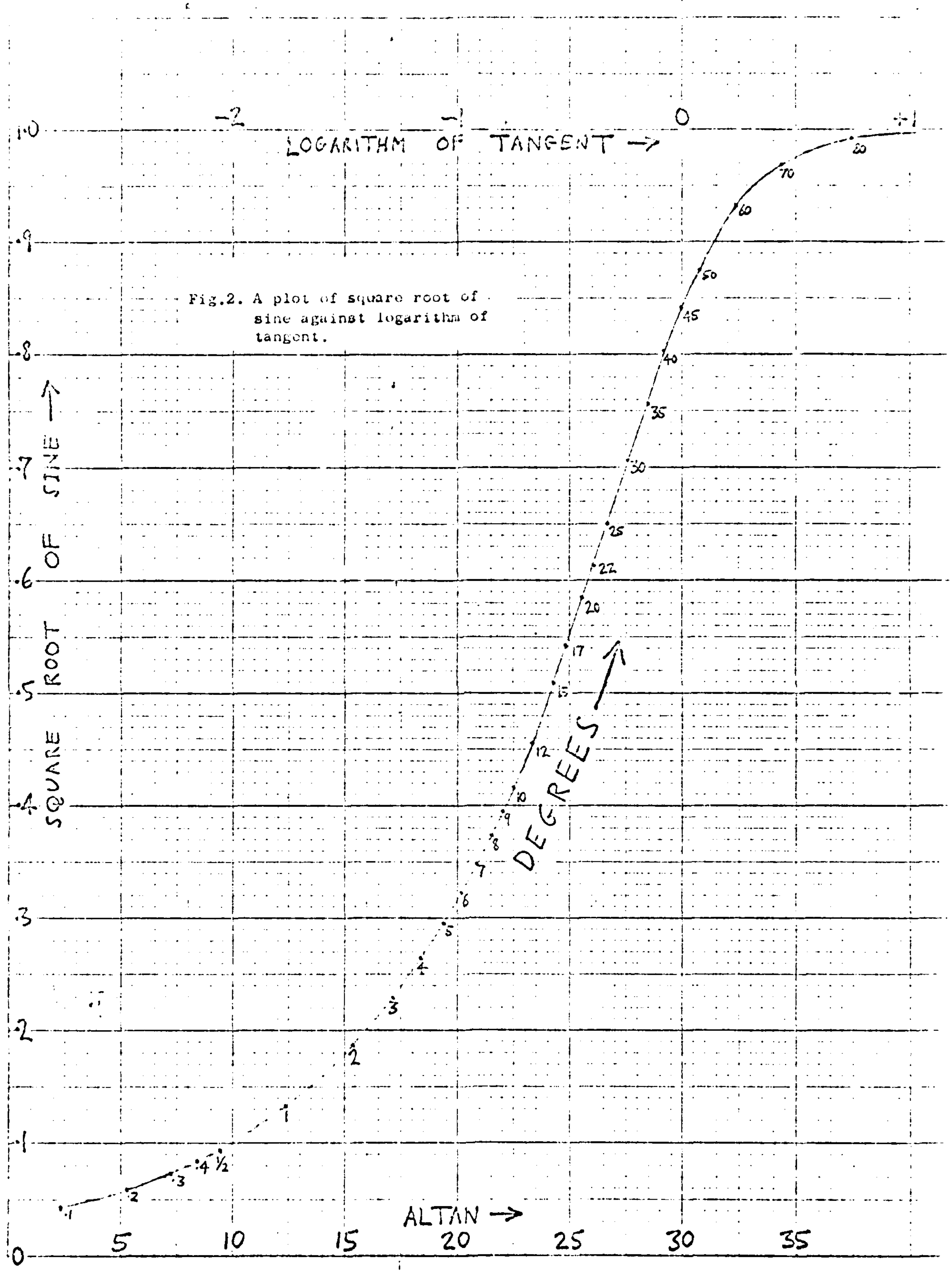

Fig.2. A plot of square root of sine against logarithm of tangent.

HISTOGRAM/FREQUENCIES

MIDPOINT°	HIST%	COUNT FOR 3.GRADIENT (EACH X= 48)
0.	2.9	245.
1.0000	55.3	4719
2.0000	28.4	2427
3.0000	9.0	772
4.0000	2.7	222
5.0000	1.1	90
6.0000	.3	26
7.0000	.2	15
8.0000	.1	6
9.0000	.0	4
10.000	0.	0
11.000	.0	2
12.000	.0	1
13.000	.0	1
14.000	.0	2
15.000	.0	1
TOTAL		8540 (INTERVAL WIDTH= 1.0000)

Fig.3. Histogram of gradients for CACHE 1, S.W. Oklahoma. The left-hand column gives class midpoint in degrees.

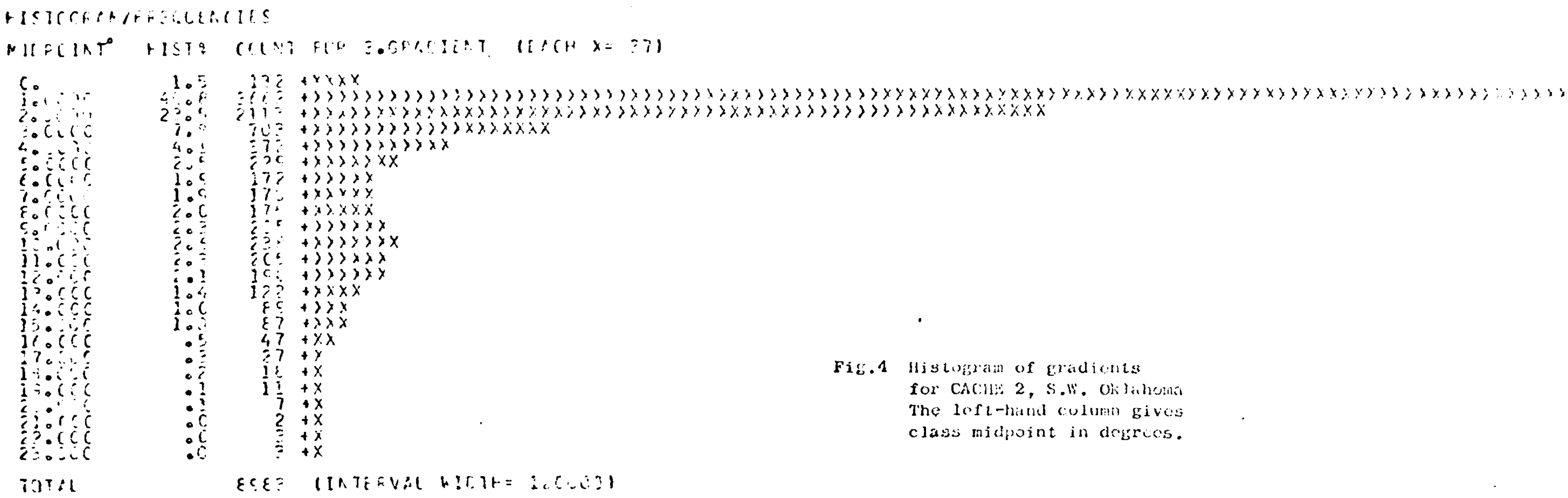

Fig.4 Histogram of gradients for CACHE 2, S.W. Oklahoma The left-hand column gives class midpoint in degrees.

HISTOGRAM/FREQUENCIES

MIDPOINT°	PCT	COUNT FOR 3.GRADIENT (EACH X= 10)
0.	.5	48
1.0000	11.3	1071
2.0000	13.6	1287
3.0000	8.9	846
4.0000	7.3	692
5.0000	8.3	789
6.0000	8.2	780
7.0000	7.4	703
8.0000	8.2	782
9.0000	7.6	720
10.0000	6.6	624
11.0000	4.9	466
12.0000	3.4	319
13.0000	1.7	16[illegible]
14.0000	.9	82
15.0000	.5	49
16.0000	.3	30
17.0000	.1	14
18.0000	.1	7
19.0000	.1	7
20.0000	0.	0
21.0000	.0	3
22.0000	.0	2
TOTAL		9481 (INTERVAL WIDTH= 1.0000)

Fig.5. Histogram of gradients for CACHE 3, S.W. Oklahoma. The left-hand column gives class midpoint in degrees.

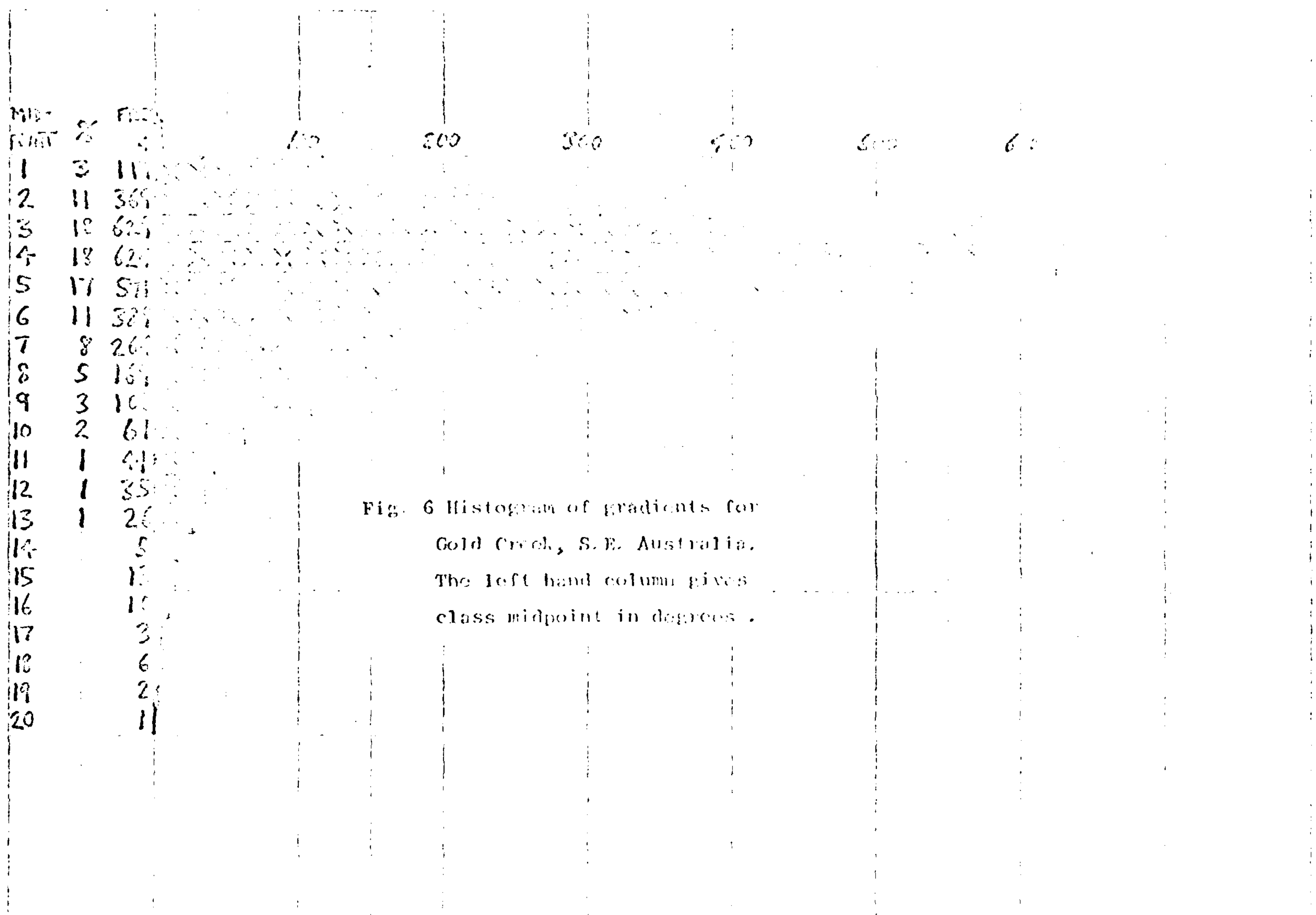

Fig. 6 Histogram of gradients for Gold Creek, S.E. Australia. The left hand column gives class midpoint in degrees .

HISTOGRAM/FREQUENCIES

MIDPOINT HIST% COUNT FOR 3,GRADIENT (EACH X= 12)

MIDPOINT	HIST%	COUNT
0.	0.	0 +
1.0000	1.1	124 +XXXXXXXXXXX
2.0000	1.2	139 +XXXXXXXXXXXX
3.0000	2.3	261 +XXXXXXXXXXXXXXXXXXXXXX
4.0000	1.5	173 +XXXXXXXXXXXXXXX
5.0000	1.8	209 +XXXXXXXXXXXXXXXXXX
6.0000	2.5	287 +XXXXXXXXXXXXXXXXXXXXXXXX
7.0000	2.5	289 +XXXXXXXXXXXXXXXXXXXXXXXXX
8.0000	3.7	425 +XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
9.0000	5.0	584 +XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
10.000	5.9	667 +XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
11.000	9.5	1135 +XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
12.000	8.0	929 +XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
13.000	9.5	1129 +XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
14.000	7.5	872 +XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
15.000	8.0	928 +XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
16.000	6.2	716 +XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
17.000	5.7	663 +XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
18.000	4.8	555 +XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
19.000	3.5	434 +XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
20.000	2.8	325 +XXXXXXXXXXXXXXXXXXXXXXXXXXX
21.000	2.2	254 +XXXXXXXXXXXXXXXXXXXXX
22.000	1.7	194 +XXXXXXXXXXXXXXXX
23.000	1.0	114 +XXXXXXXXXX
24.000	.8	85 +XXXXXXXX
25.000	.4	52 +XXXXX
26.000	.3	35 +XXX
27.000	.3	29 +XXX
28.000	.1	10 +X
29.000	.1	15 +XX
30.000	.1	7 +X
31.000	0.	0 +
32.000	.0	3 +X
33.000	.0	2 +X
34.000	.0	2 +X
35.000	.0	2 +X

TOTAL [illegible] (INTERVAL WIDTH= 1.0000)

Fig. 7 Histogram of gradients for Ferro, South Italy. The left-hand column gives class midpoint in degrees.

Fig. 8 Histogram of gradients for Nupur, N.W. Iceland. The left-hand column gives class midpoint in degrees.

Fig.9 Histogram of gradients for TORRIDON, N.W. Scotland. The left-hand column gives class midpoint in degrees.

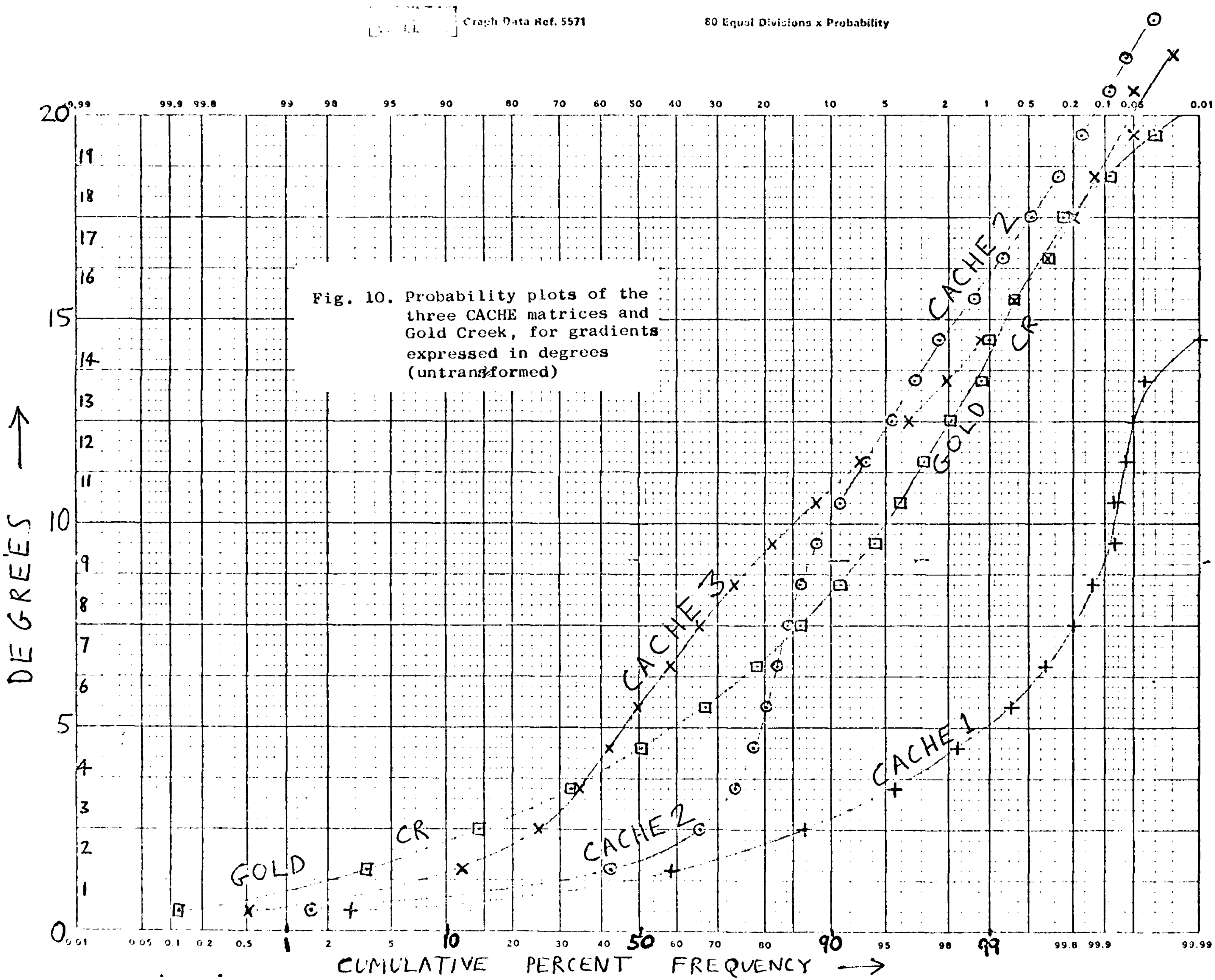

Fig. 10. Probability plots of the three CACHE matrices and Gold Creek, for gradients expressed in degrees (untransformed)

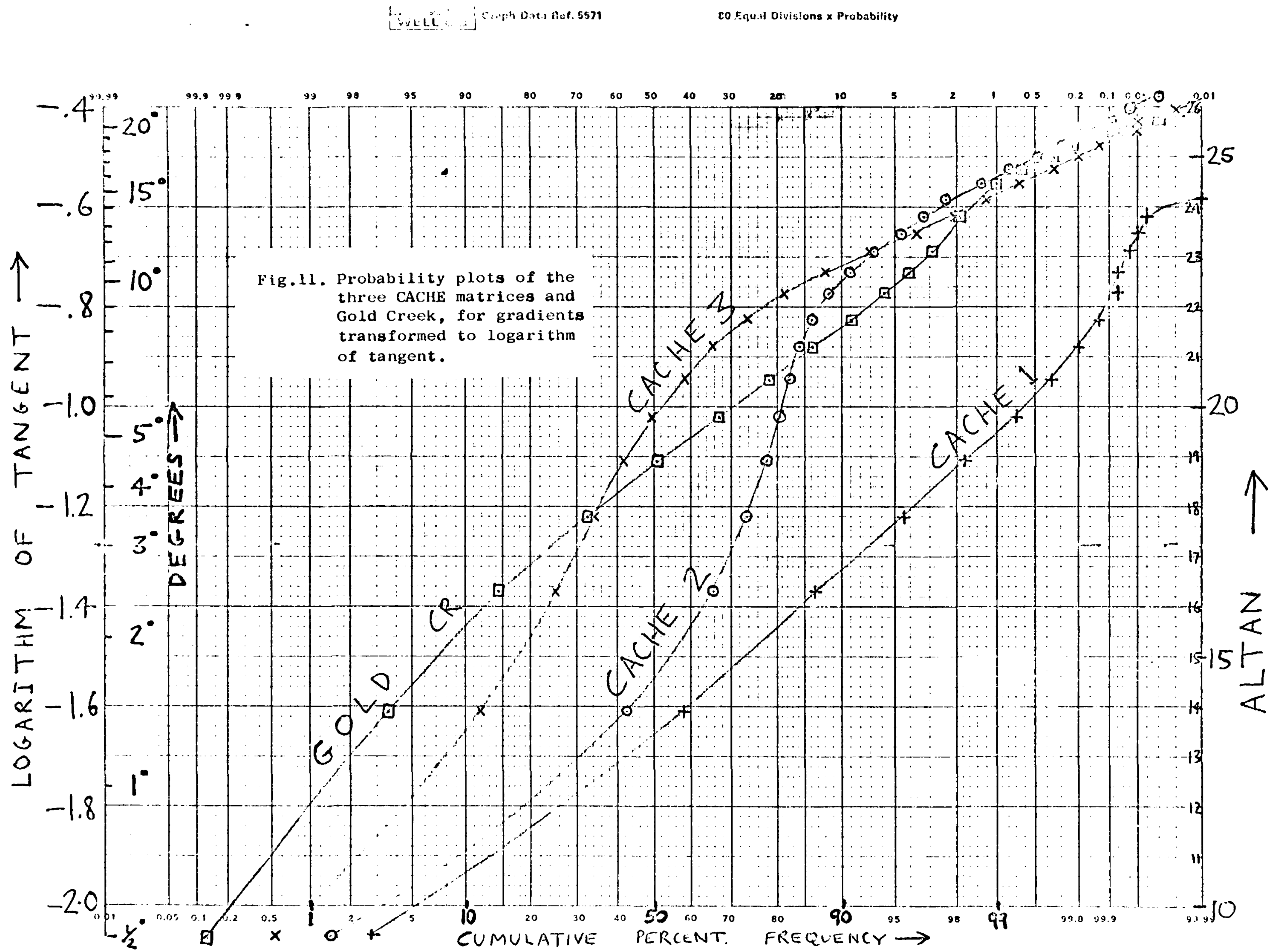

Fig.11. Probability plots of the three CACHE matrices and Gold Creek, for gradients transformed to logarithm of tangent.

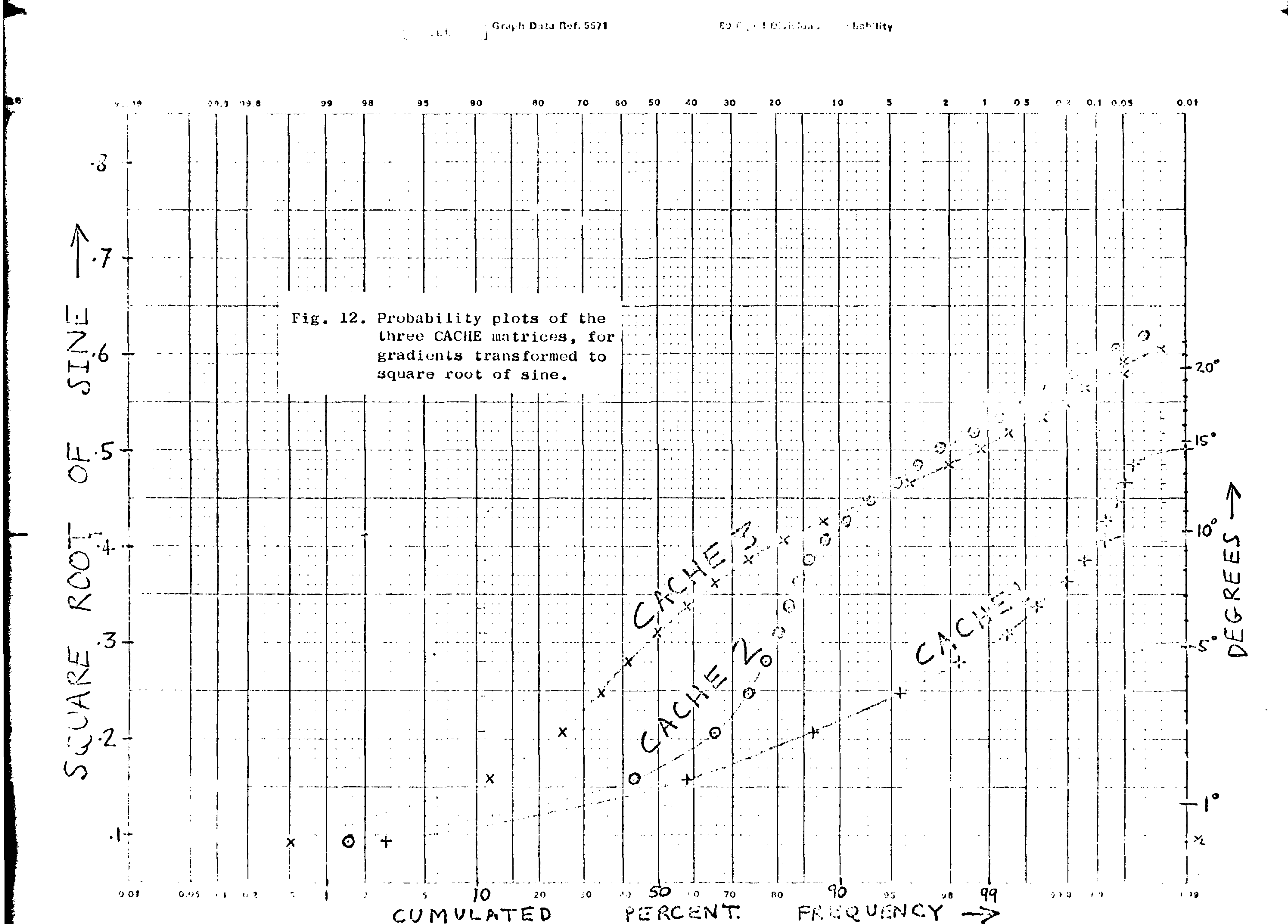

Fig. 12. Probability plots of the three CACHE matrices, for gradients transformed to square root of sine.

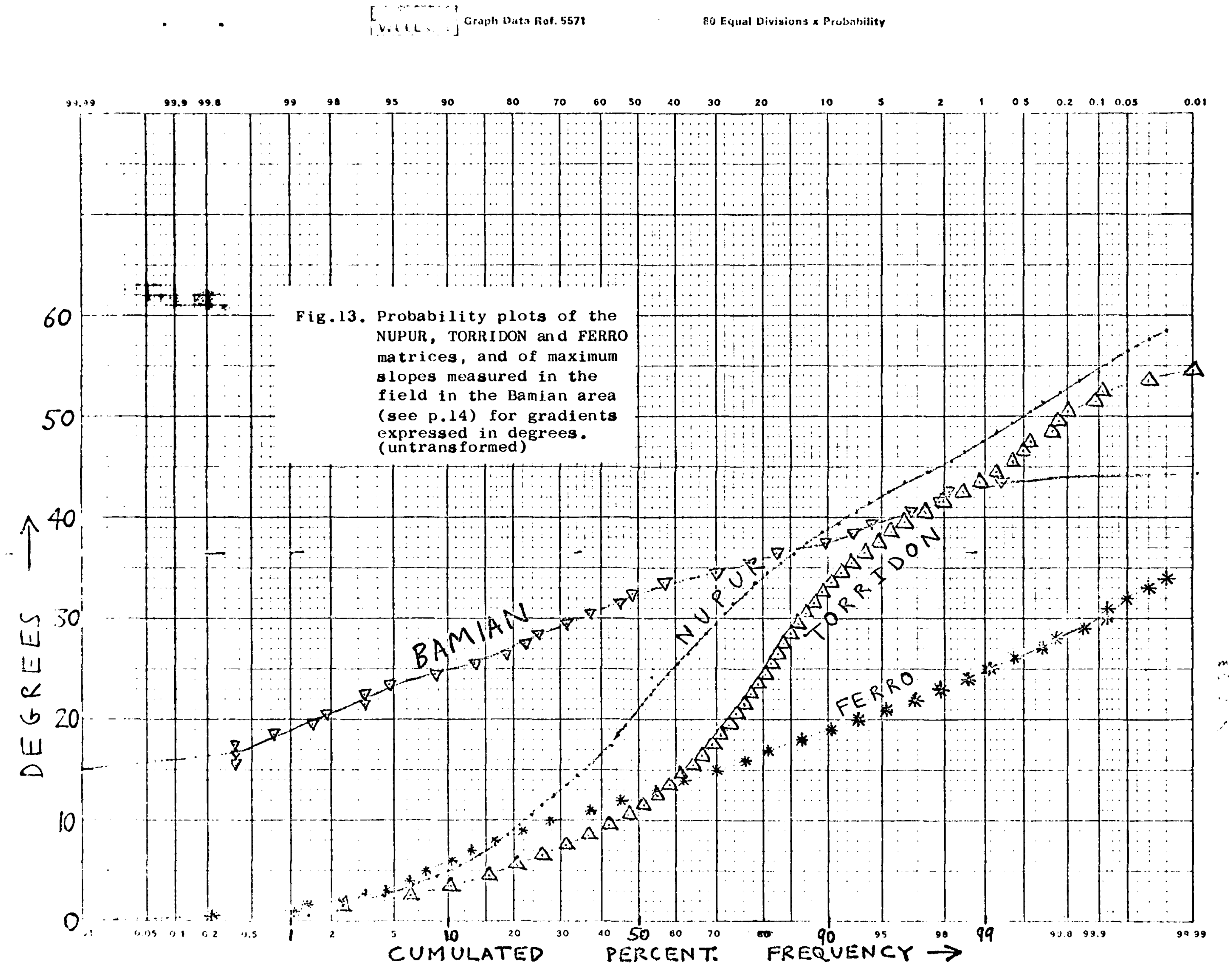

Fig.13. Probability plots of the NUPUR, TORRIDON and FERRO matrices, and of maximum slopes measured in the field in the Bamian area (see p.14) for gradients expressed in degrees. (untransformed)

Graph Data Ref. 5571 80 Equal Divisions x Probability

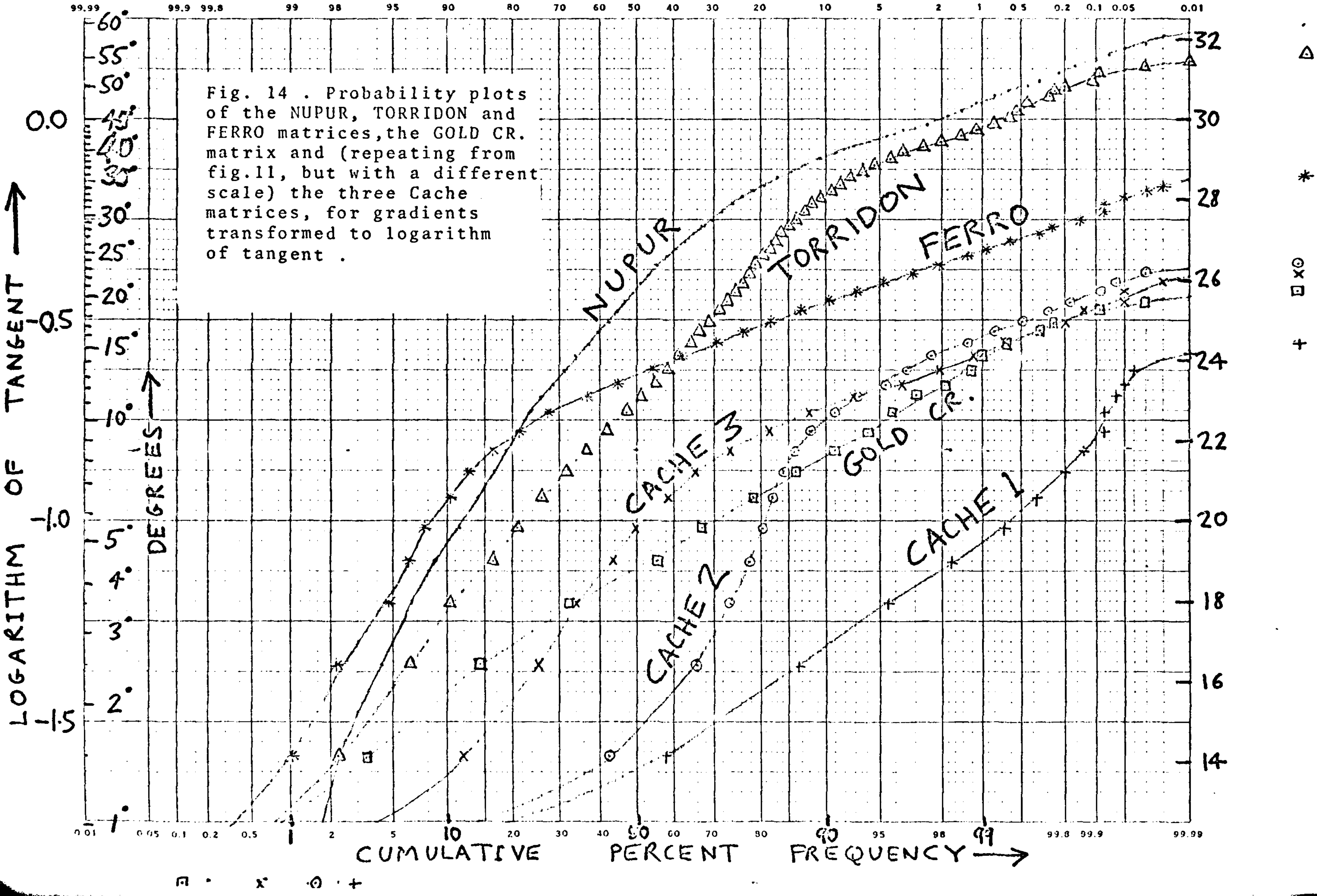

Fig. 14 . Probability plots of the NUPUR, TORRIDON and FERRO matrices, the GOLD CR. matrix and (repeating from fig.11, but with a different scale) the three Cache matrices, for gradients transformed to logarithm of tangent .

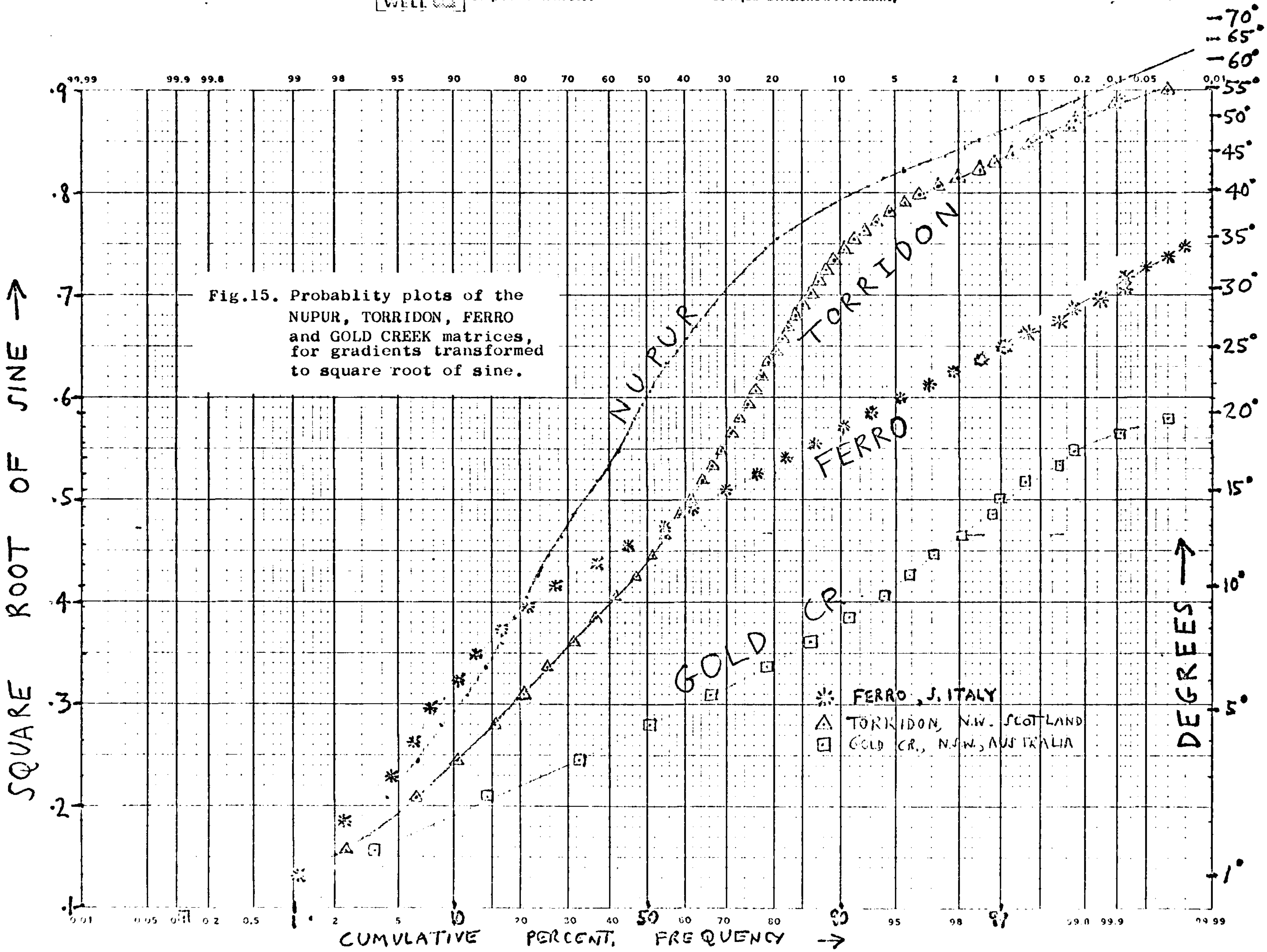

Fig.15. Probablity plots of the NUPUR, TORRIDON, FERRO and GOLD CREEK matrices, for gradients transformed to square root of sine.

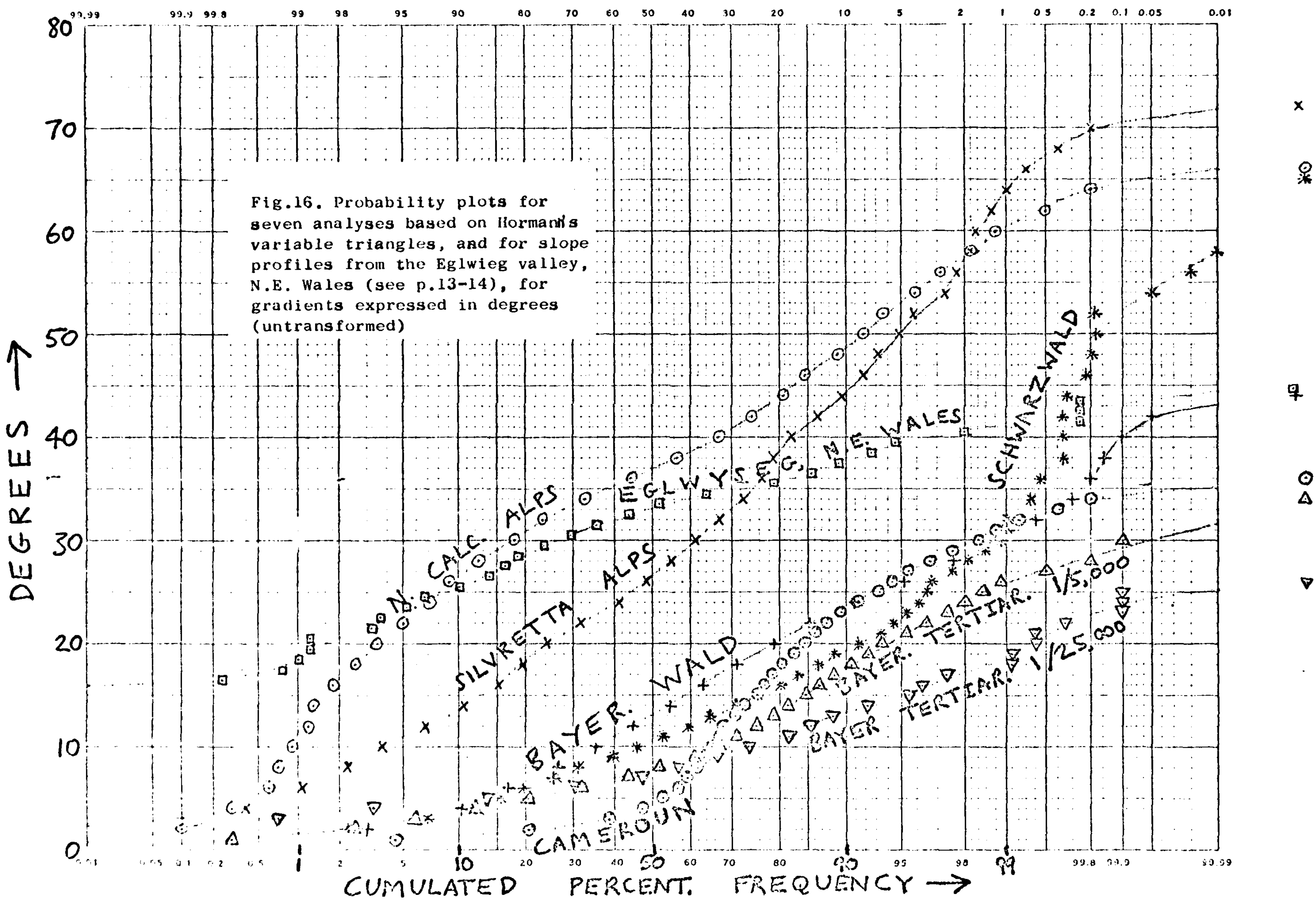

Fig.16. Probability plots for seven analyses based on Hormann's variable triangles, and for slope profiles from the Eglwieg valley, N.E. Wales (see p.13-14), for gradients expressed in degrees (untransformed)

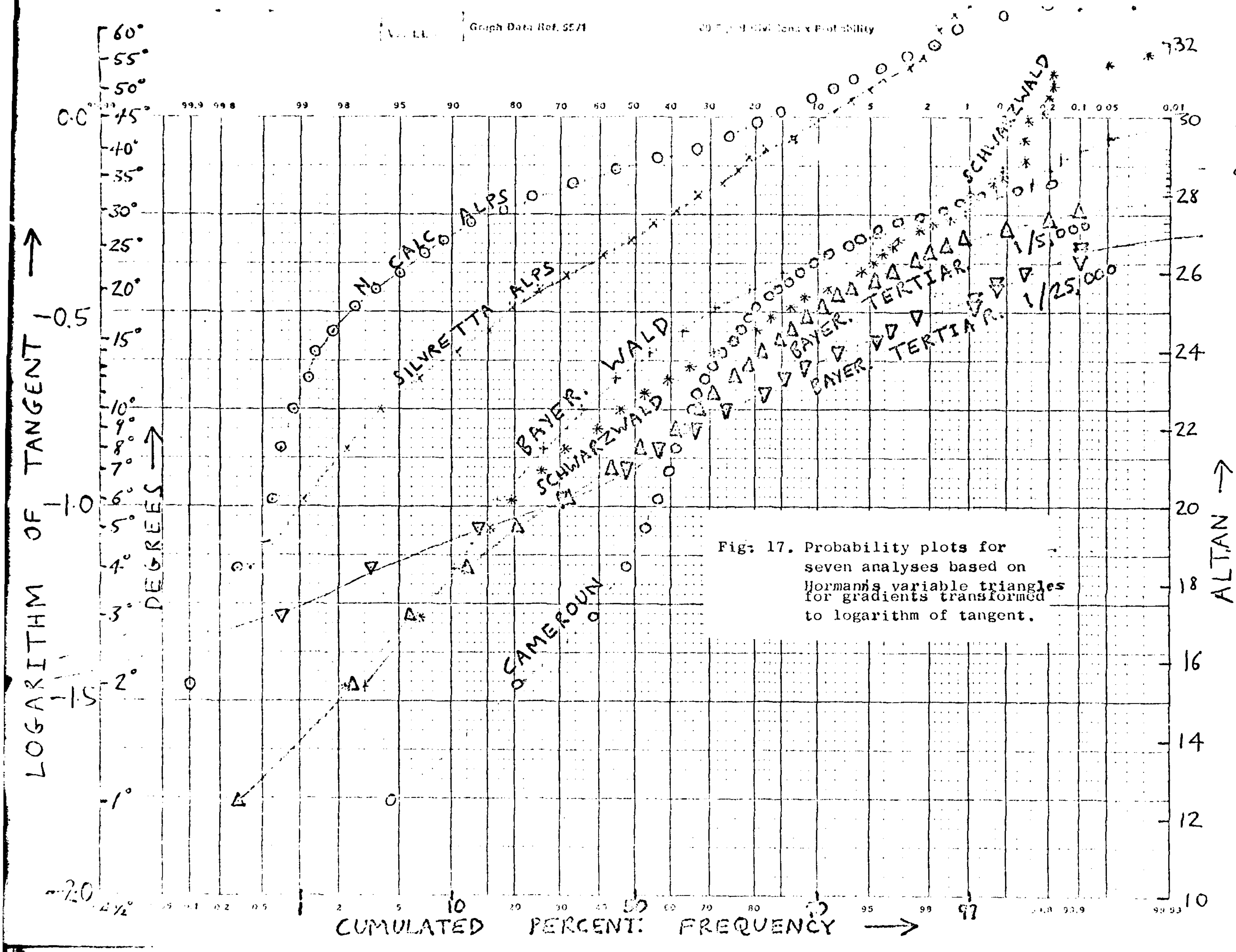

Fig. 17. Probability plots for seven analyses based on Hormann's variable triangles for gradients transformed to logarithm of tangent.

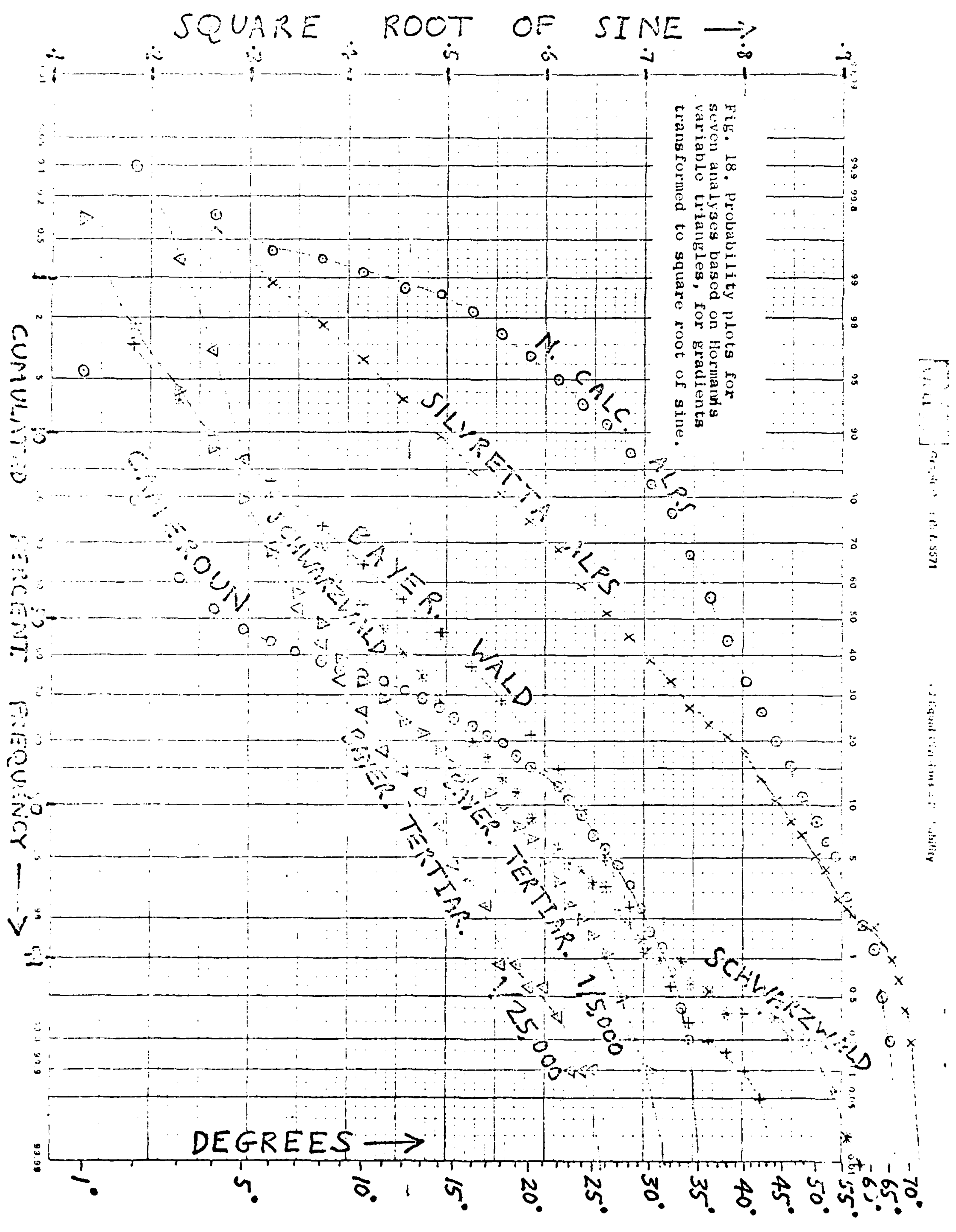

Fig. 18. Probability plots for seven analyses based on Hormann's variable triangles, for gradients transformed to square root of sine.

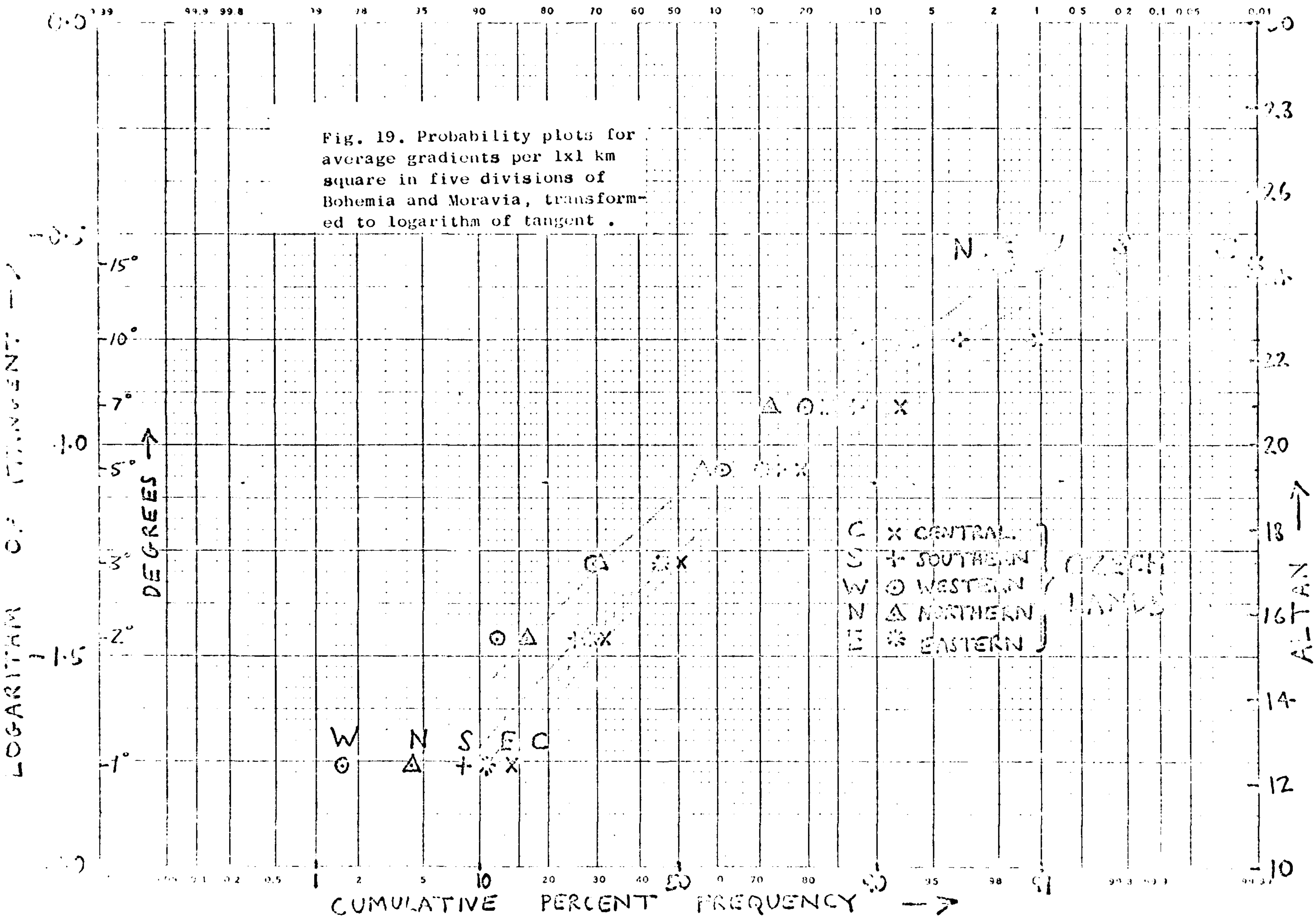

Fig. 19. Probability plots for average gradients per 1x1 km square in five divisions of Bohemia and Moravia, transformed to logarithm of tangent .

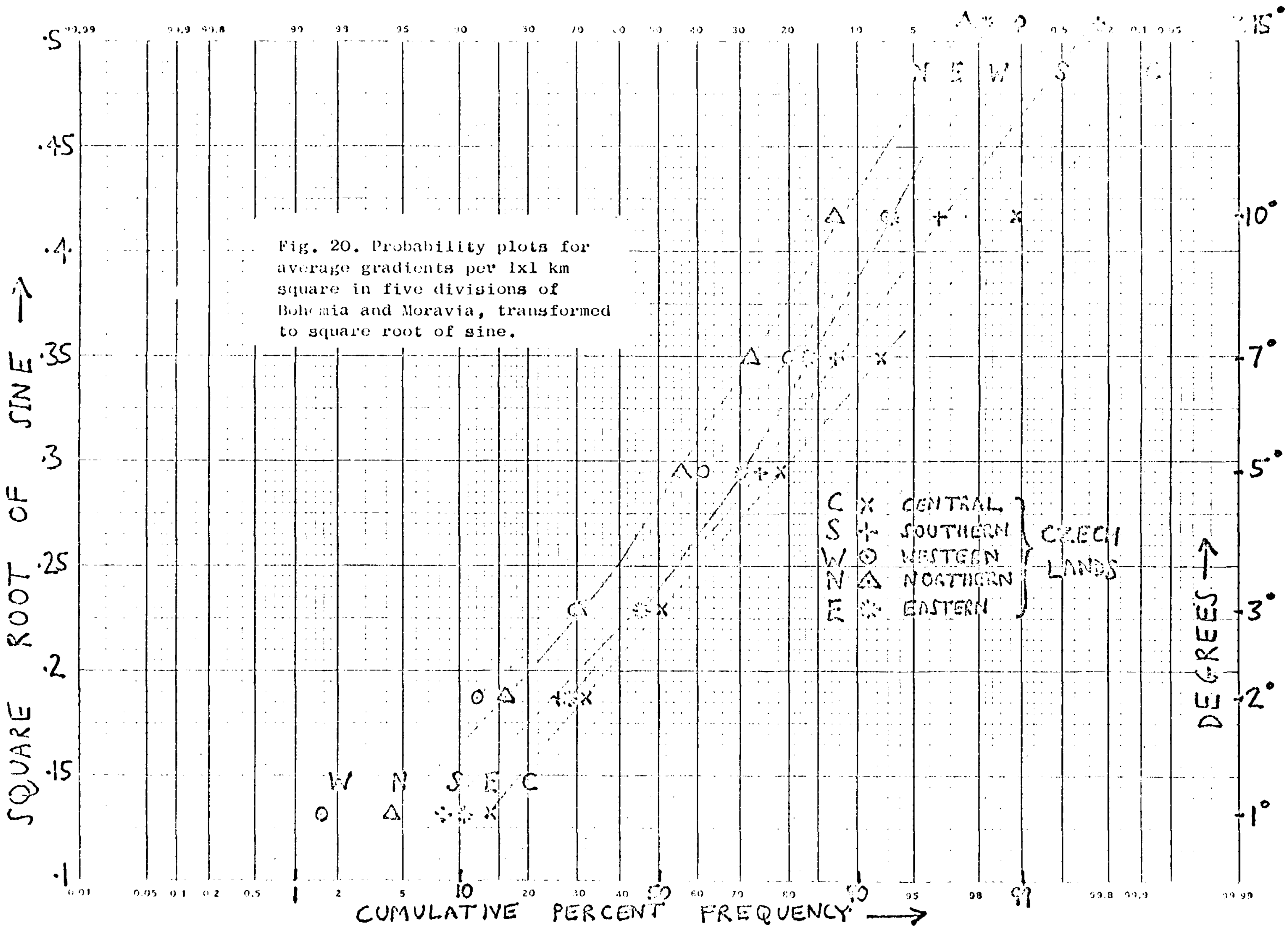

Fig. 20. Probability plots for average gradients per 1x1 km square in five divisions of Bohemia and Moravia, transformed to square root of sine.

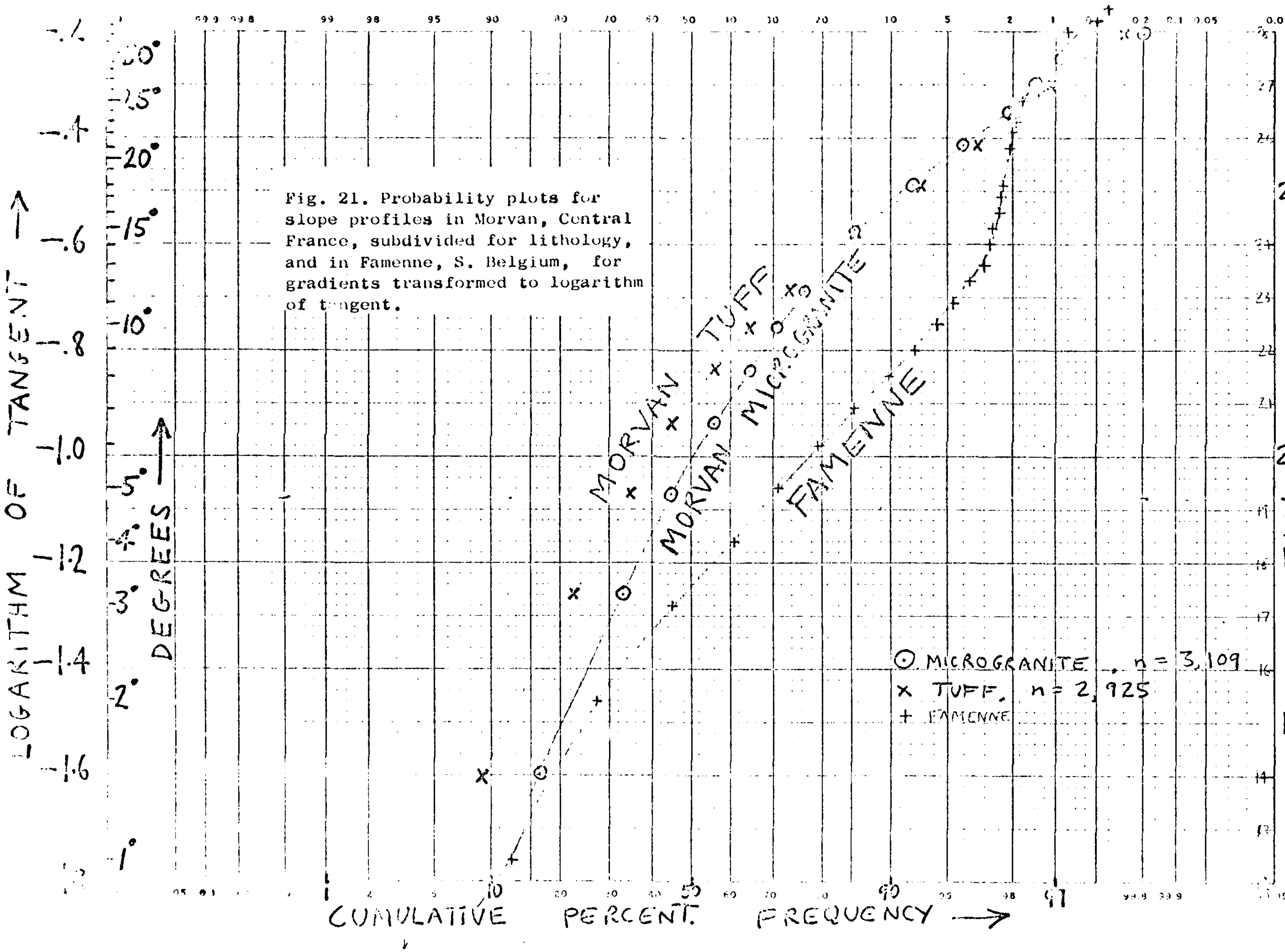

Fig. 21. Probability plots for slope profiles in Morvan, Central France, subdivided for lithology, and in Famenne, S. Belgium, for gradients transformed to logarithm of tangent.

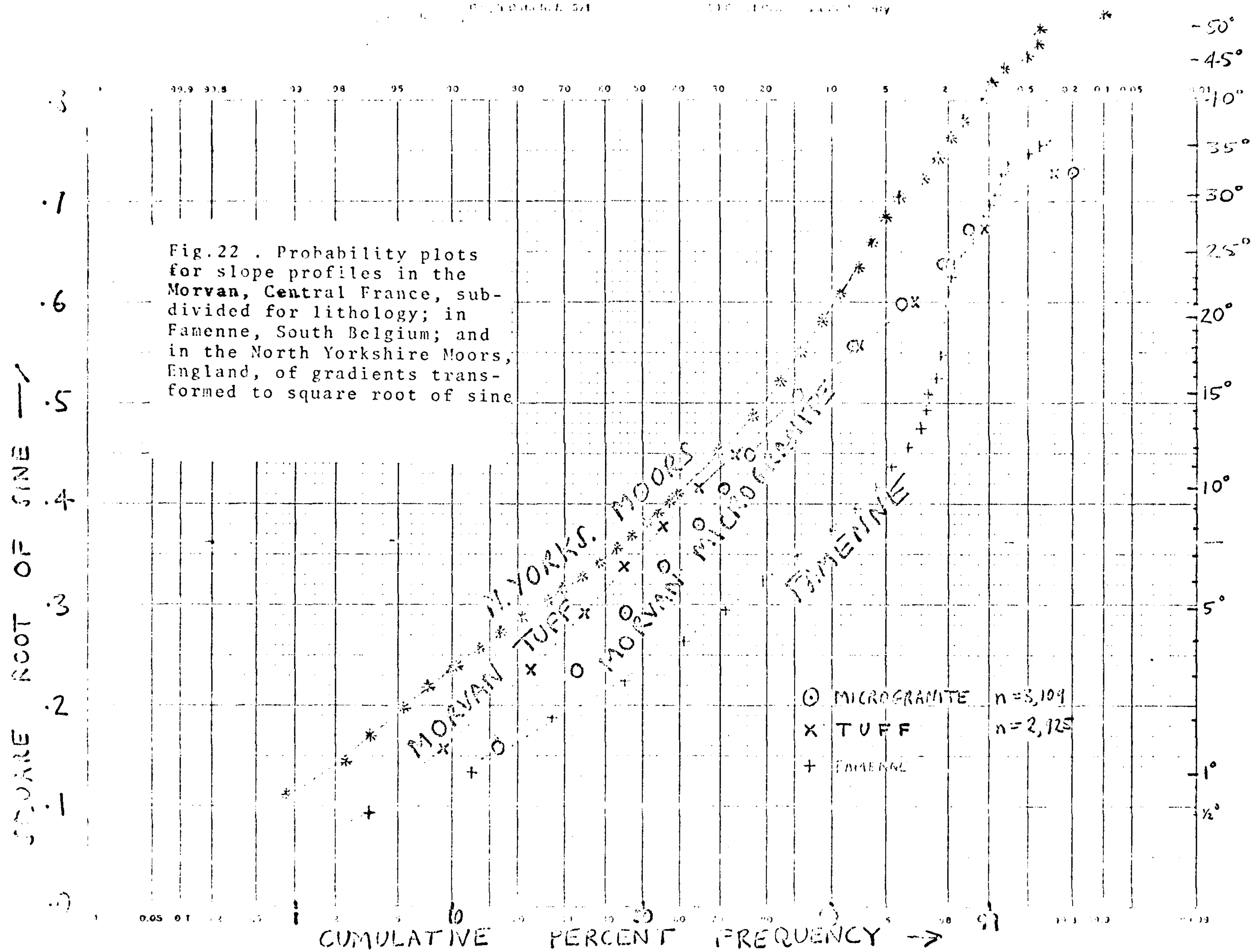

Fig.22 . Probability plots for slope profiles in the **Morvan, Central France**, subdivided for lithology; in Famenne, South Belgium; and in the North Yorkshire Moors, England, of gradients transformed to square root of sine

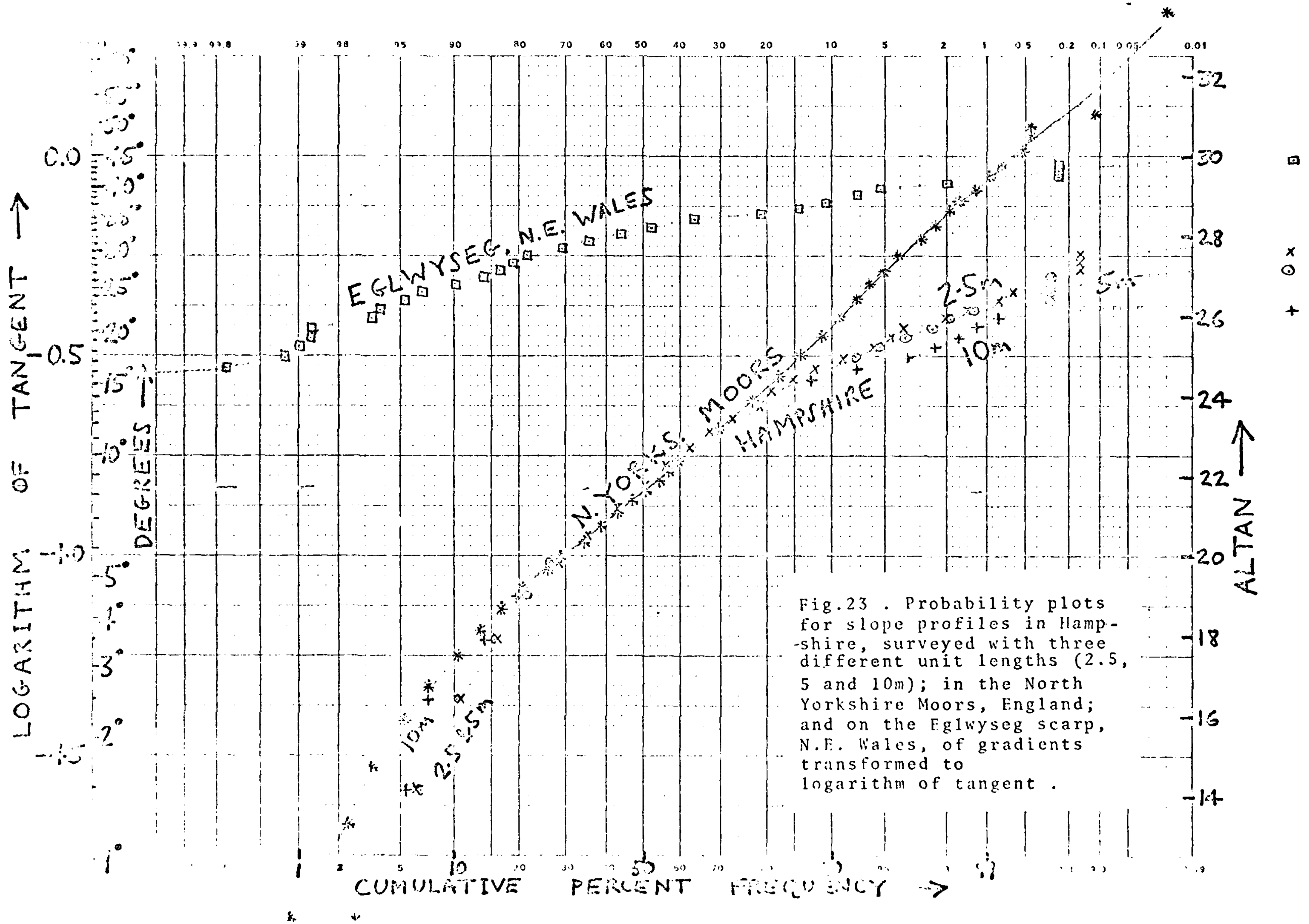

Fig.23 . Probability plots for slope profiles in Hampshire, surveyed with three different unit lengths (2.5, 5 and 10m); in the North Yorkshire Moors, England; and on the Eglwyseg scarp, N.E. Wales, of gradients transformed to logarithm of tangent .

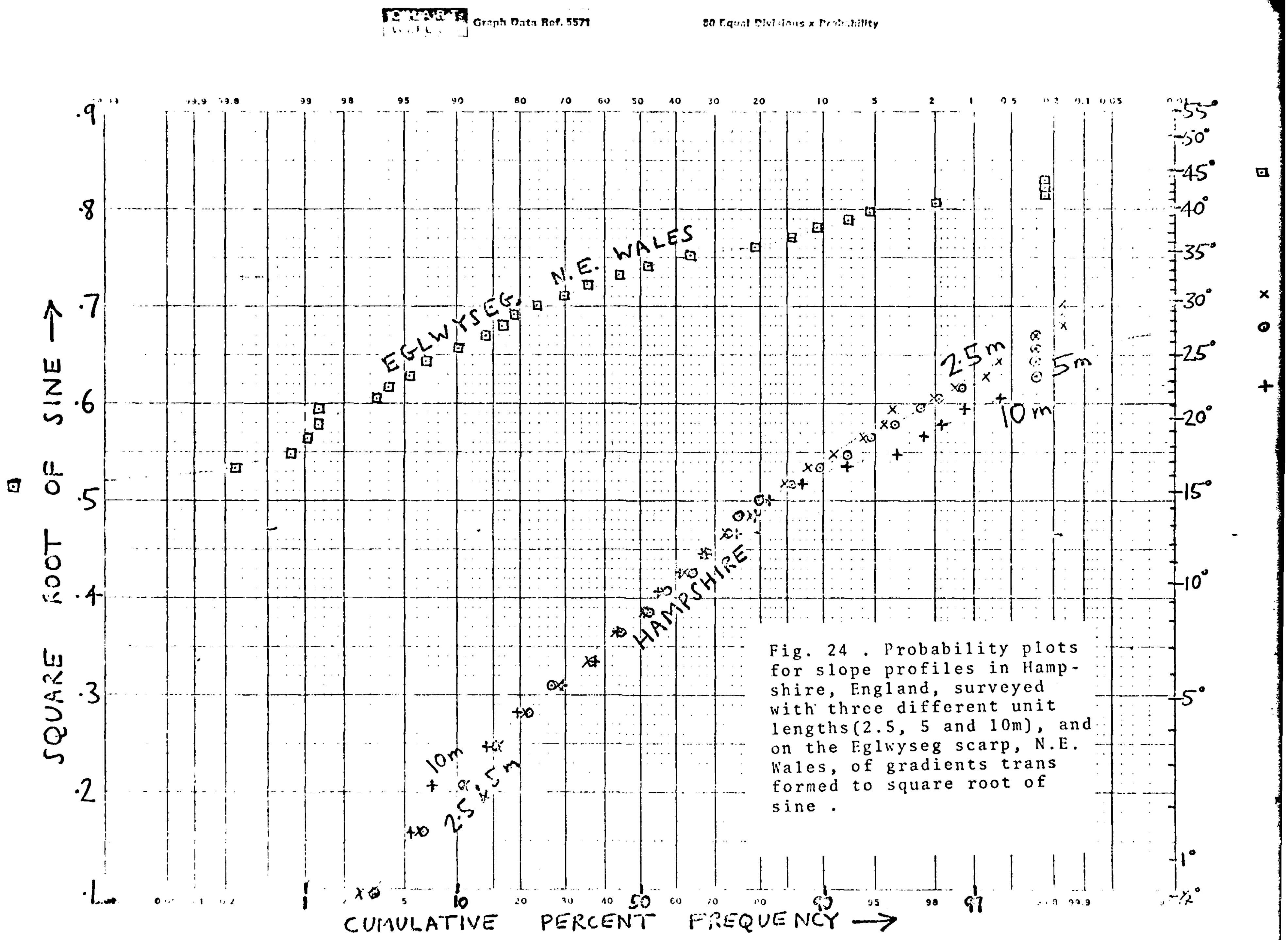

Fig. 24 . Probability plots for slope profiles in Hampshire, England, surveyed with three different unit lengths(2.5, 5 and 10m), and on the Eglwyseg scarp, N.E. Wales, of gradients trans formed to square root of sine .